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Effects of Dephasing and Dissipation on Nonequilibrium Quantum Noise

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Nonequilibrium current fluctuations in a conductor in the presence of transport are studied. When the conductor length L is of mesoscopic size, this nonequilibrium noise is of quantum nature; it increases with the current and does *not* vanish even at zero temperature. As L is increased beyond the dephasing length, the quantum coherence is destroyed, but the noise is *not* suppressed; it increases toward the shot noise level. This is in sharp contrast with other quantum phenomena such as universal conductance fluctuations, which are suppressed by dephasing. As L is further increased to the length of the maximum energy-relaxation, the noise starts to decrease, approaching zero in the macroscopic limit. Maximum energy dissipation is therefore essential for the noise to be of macroscopic type. A scaling theory connecting mesoscopic and macroscopic regimes is also presented.
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I. INTRODUCTION

Electrical current J in a conductor fluctuates about the mean value $\langle J \rangle$ as a function of time. At thermal equilibrium, the fluctuations (henceforth called "noise") $\langle \delta J^2 \rangle$ are related to the linear conductivity by the fluctuation-dissipation theorem. In the presence of transport, $\langle J \rangle \neq 0$, however, a *nonequilibrium* noise (NEN) appears [1–3], which in general has no simple relationship to the transport coefficients. At low temperatures, the equilibrium noise becomes negligible, whereas the NEN increases with current [1, 2]. Throughout this work, we assume zero temperature, so that the total noise is equal to the NEN. This NEN has significant effects on many physical systems. For example, when a conductor is connected in series to a light-emitting (or laser) diode of high efficiency, a sub-Poissonian (non-classical) photon state can be generated if the NEN of the conductor is smaller than the shot noise [4]. The NEN also determines the fundamental performance limits of quantum interference devices [2].

Despite being so important, the formula for the NEN has only been obtained for two limiting cases: for mesoscopic conductors with perfect quantum coherence [1, 2] and for macroscopic conductors [5]. That is, for a one-dimensional mesoscopic conductor connected to ideal reservoirs at zero temperature [1],

$$W = 1 - T \quad \text{for mesoscopic conductors,} \tag{1}$$

where T is the transmittance determined by potential scatterers in the conductor [6], and W is the "noise figure" [7] defined by

$$W \equiv \langle \delta J^2 \rangle / \langle \delta J^2 \rangle_{\text{shot}}. \tag{2}$$

Here, $\langle \delta J^2 \rangle_{\text{shot}}$ is the usual shot noise, which is proportional to $\langle J \rangle$. Clearly, for a given chemical potential difference $\Delta\mu$, $\langle \delta J^2 \rangle$ decreases with decreasing $\langle J \rangle$. The noise figure has been introduced to cancel out this trivial dependence on the samples' conductivities [7], and it corresponds to the Fano factor, which is the standard measure of quantum noise of photons [8]. Equation (1) neglects the $1/f$ noise because for samples of good quality it is negligible except at very low frequencies [9].

If the length L of the conductor is increased, T eventually goes to zero and W approaches unity, i.e., Eq. (1) predicts that the NEN approaches the full shot noise:

$$W \to 1 \quad as \quad L \to \infty. \tag{3}$$

It is well known [5], however, that

$$W \simeq 0 \quad \text{for macroscopic conductors.} \tag{4}$$

In this work, we resolve this apparent contradiction by analyzing the effects of non-ideal reservoirs, dephasing, and dissipation on NEN, and we clarify the mechanism that distinguishes macroscopic conductors from mesoscopic conductors [7].

Note that the NEN of Eq. (1) is purely of quantum nature, and it is therefore tempting to say that the NEN should be suppressed when quantum coherence is destroyed by a dephasing mechanism. However, this is *false*, as we will show later. The point is that not only a quantum system, but also a classical (incoherent) system can exhibit noise. This is in sharp contrast with other quantum phenomena such as universal conductance fluctuations [10], which are suppressed when quantum coherence is destroyed.

II. NON-IDEAL RESERVOIRS

We consider a one-dimensional conductor of length L, both ends of which are connected to large electron reservoirs 1 and 2, whose chemical potentials are μ_1 and μ_2 ($\leq \mu_1$), respectively. Current J is induced by the chemical potential difference between these two reservoirs: $\Delta\mu \equiv \mu_1 - \mu_2$. Equation (1) assumes that the reservoirs are ideal. In real samples with nonzero $\Delta\mu$, however, the reservoirs (particularly their boundaries on the conductor) could be excited by a finite current. If the energies of excited electrons are smaller than $\Delta\mu$, we find – assuming perfect coherence in the conductor – that [11]

$$W = \eta(\Delta\mu)T + (1 - T), \tag{5}$$

where $0 \leq \eta(\Delta\mu) \leq 1$ measures the degree of the excitation, the detailed form of which is irrelevant to the present discussion. The incompleteness of the reservoirs causes an additional noise, ηT, which is the scaled emission noise [2, 12]. This formula explains why the W observed for a quantum point contact [9] does not vanish when $T = 1$, where Eq. (1) predicts $W = 0$. However, it predicts, as Eq. (1) does, the asymptotic behavior of Eq. (3). Hence, the imperfectness of reservoirs does *not* lead to Eq. (4).

III. A GENERAL MICROSCOPIC MODEL

Next, we consider dephasing and dissipation *in the conductor*, assuming that the reservoirs are ideal. Equation (1) was derived by neglecting any interactions of electrons with other electrons, or with phonons, photons, or magnetic impurities, etc. When these interactions are taken into account, the lifetime $\eta_{\rm ife}$ of a one-body electron state becomes finite. For mesoscopic conductors, however, a more important time scale is the phase relaxation time τ_ϕ, which is usually longer than $\eta_{\rm ife}$ [10]. We will show later that there is another important time scale $\tau_{\rm rlx}$, which is the time spent by an electron of energy $\Delta\mu$ above the Fermi energy before it relaxes onto the Fermi surface [see Eqs. (9), (13) and (14)]. Note that $\tau_{\rm rlx}$ is generally longer than a simple inelastic lifetime: Two or more inelastic-scattering events occur before an electron loses all excess energy of $\Delta\mu$. That is, $\tau_{\rm rlx}$ is the time for the maximum energy dissipation.

To demonstrate that the dephasing process plays a role completely different from this energy-relaxation process, we employ a model in which τ_ϕ can be shortened while keeping $\tau_{\rm rlx}$ long. That is, we consider conductors with magnetic impurities and electron-phonon interactions [7]. We denote by $\hat{V}_{em}$ the sum of interactions between electrons and the magnetic impurities, each of which is assumed to be weak and ferromagnetic so that the pure-dephasing condition is ideal [13]. As the concentration of the magnetic impurities is increased, the spin of an electron will be flipped by the impurities. This disturbs the electron interference, so τ_ϕ is shortened while the energy of the electron is preserved. On the other hand, the electron-phonon interaction $\hat{V}_{ep}$ induces phonon-emission processes and thereby determines $\tau_{\rm rlx}$.

Following the standard technique [6], we suppose that perfect leads 1 and 2 are connected to the conductor of length L. After lengthy calculations [7], we find a general formula for W, which at zero temperature takes the very simple form:

$$W \simeq \frac{\sum_\epsilon [1 - \tilde{f}_\alpha(\epsilon)]\tilde{f}_\alpha(\epsilon)}{\sum_\epsilon [f_\alpha(\epsilon) - \tilde{f}_\alpha(\epsilon)]}, \tag{6}$$

where $f_\alpha(\epsilon)$ and $\tilde{f}_\alpha(\epsilon)$ are the distribution functions of in-coming and out-going electrons, respectively, in lead α $(= 1, 2)$. Let us confirm that in the coherent regime $(L << L_\phi, L_{\rm rlx})$ this formula reproduces the previous results. Since the reservoirs are assumed to be ideal, $f_\alpha(\epsilon)$ obeys the perfect Fermi-Dirac (FD) distribution. On the other hand, when $\hat{V}_{ep} = \hat{V}_{em} = 0$, $\tilde{f}_1(\epsilon)$ is given by [7]

$$\tilde{f}_1(\epsilon) = \tilde{f}_1^0(\epsilon) \equiv \begin{cases} 0 & (\mu_1 < \epsilon), \\ 1 - T & (\mu_2 < \epsilon \leq \mu_1), \\ 1 & (\epsilon \leq \mu_2). \end{cases} \tag{7}$$

If we neglect a weak ϵ dependence of T for $\mu_2 < \epsilon \leq \mu_1$, then Eqs. (6), (7) indeed reproduce Eq. (1).

A. EFFECT OF DEPHASING

Most quantum phenomena that are characteristic of mesoscopic conductors disappear when the transit time $\tau_{\rm tr}$ of an electron through the conductor exceeds τ_ϕ [6, 10]. It is therefore worthwhile to examine whether W is suppressed when $\tau_\phi < \tau_{\rm tr} << \tau_{\rm rlx}$. This is equivalent, in length scale, to $L_\phi < L << L_{\rm rlx}$. Our model yields the last inequality $(L << L_{\rm rlx})$ when $\hat{V}_{ep} = 0$. In this case, we find that $\tilde{f}_1$ takes the same functional form as Eq. (7), but we should replace T with a generalized transmittance. That is, since the electron spin is not conserved, T can no longer be defined as the square of the usual one-body scattering amplitude. Instead, we define T simply as the probability that an electron passes through the conductor, disregarding whether or not its spin is conserved. With this T, $\tilde{f}_1$ is given by Eq. (7), and therefore

$$W \simeq 1 - T \qquad (\text{for any } L << L_{\rm rlx}). \tag{8}$$

That is, although Eq. (1) was obtained for $L << L_\phi, L_{\rm rlx}$ the same *form* approximately holds even when $L > L_\phi$ if the generalized transmittance is used for T. (The *value* of this T is of course different from that for $\hat{V}_{em} = 0$.) As

L is increased, this T eventually goes to zero. Hence, we see that the dephasing without energy relaxation does *not* lead to Eq. (4). This is in sharp contrast to other quantum phenomena in mesoscopic systems, such as the universal conductance fluctuations, most of which are suppressed by the dephasing [6, 10].

B. EFFECT OF DISSIPATION

We now consider the energy dissipations of electrons by taking $\hat{V}_{ep} \neq 0$ and $\hat{V}_{em} = 0$. Let us first consider the damped limit where $\hat{V}_{ep}$ is very large. In this limiting case, all out-going electrons are relaxed to states of minimum allowable energy, so

$$\tilde{f}_\alpha(\epsilon) = \tilde{f}_\alpha^\infty(\epsilon) \equiv \begin{cases} 0 & (\tilde{\mu}_\alpha < \epsilon), \\ 1 & (\epsilon \leq \tilde{\mu}_\alpha). \end{cases} \tag{9}$$

Here, $\tilde{\mu}_\alpha$ $(\alpha = 1, 2)$ denotes the Fermi energy of out-going electrons in lead α. This quantity can be defined only in the damped limit. Substituting this form into Eq. (6), we get

$$W = 0 \qquad \text{when} \quad L/L_{\rm rlx} \to \infty, \tag{10}$$

which agrees with Eq. (4). Equations (5), (8), and (10) answer our question: The maximum energy transfer from the electron system to the phonon system (or photon system [14]) is essential for noise suppression.

To see this in more detail, let us interpolate between Eq. (8) and Eq. (10). As L is increased from $L << L_\phi$ toward $L >> L_{\rm rlx}$, $\tilde{f}_\alpha$ gradually changes from $\tilde{f}_\alpha^0$ to $\tilde{f}_\alpha^\infty$. Hence, we can conclude that W changes continuously from Eq. (8) to Eq. (10). To see the explicit functional form of W, we have developed a simple microscopic theory [14] and a scaling theory [11]. We present below the essence of these theories.

IV. A SIMPLE MICROSCOPIC MODEL

To calculate the explicit functional form of W, we must specify the forms of $\hat{V}_{em}$ and $\hat{V}_{ep}$. For simplicity, we take $\hat{V}_{em} = 0$, and

$$\hat{V}_{ep} = \sum_{k,q} \hbar g_{kq} \hat{a}_k^\dagger \hat{a}_k (\hat{c}_q + \hat{c}_q^\dagger), \tag{11}$$

where $\hat{c}_q$ is the annihilation operator of phonons, and g_{kq} is the coupling constant of the electron-phonon interaction [14]. Although very simple, this model seems to describe the point. By a unitary transformation, the Hamiltonian can be further simplified, and we finally obtain the formula [14],

$$W \simeq (1 - \kappa)(1 - T), \tag{12}$$

$$\kappa \equiv \frac{1}{\Delta\mu} \int EP(E)\, dE, \tag{13}$$

where $P(E)$ denotes the phonon excitation spectra of the coupled electron-phonon system, and κ represents the average energy transfer, per transit of one electron, from the electron system to the phonon system. That is, $0 \leq \kappa \leq 1$, and κ increases with increasing $\hat{V}_{ep}$ or L, in agreement with the conclusion of the previous section.

To relate κ with $\tau_{\rm tr}$, we may employ, as a first approximation, the Markovian approximation. That is,

$$\kappa \simeq 1 - \exp[-\tau_{\rm tr}/\tau_{\rm rlx}]. \tag{14}$$

To see the L dependence, we note that the mean-free path ℓ for momentum relaxation processes of an electron is usually smaller than $L_{\rm rlx}$. When $L \sim L_{\rm rlx} >> \ell$, an electron undergoes diffusive motions, going back and forth in the conductor until it escapes into a lead. In this case, $\tau_{\rm tr}$ is proportional to L^2 for a given $\hat{V}_{ep}$, and we can rewrite Eq. (14) as $\kappa \simeq 1 - \exp[-(L/L_{\rm rlx})^2]$. We thus obtain the approximate L dependence,

$$W \simeq e^{-(L/L_{\rm rlx})^2}(1 - T) \quad (\ell << L \sim L_{\rm rlx}). \tag{15}$$

This formula explicitly shows the importance of the energy relaxation processes. We have also evaluated the frequency dependence of the NEN, which will be described elsewhere [14].

V. SCALING THEORY

The above model illustrates the dependence of the NEN on dissipation for any L. However, since it starts from a specific form of $\hat{V}_{ep}$, the resultant equation (12) should be considered to describe a qualitative dependence of the NEN on dissipation. Equation (15) is more approximate, and will not hold in such regimes that deviations from Eq. (4) or Eq. (8) are small. The scaling theory presented below plays a complementary role: It holds for *any* sample, but, at present, only small deviations from Eq. (4) or Eq. (8) can be treated.

We note that W is expected to become less sensitive to the details of the sample for $L > L_\phi$ than for $L < L_\phi$. In fact, let

$$\beta \equiv \frac{\partial \ln W}{\partial \ln x}, \quad x \equiv \frac{L}{L_{\rm cr}} \tag{16}$$

where $L_{\rm cr}$ ($\sim L_{\rm rlx}$) is the crossover length between mesoscopic and macroscopic behaviors. We have found that this β has *universal* forms in both mesoscopic and macroscopic regimes [11]:

$$\beta = \begin{cases} 1 - W \geq 0 & (x_\phi < x << 1), \\ -1 \quad\; < 0 & (1 << x). \end{cases} \tag{17}$$

We can immediately obtain the universal L dependence of W by integrating this equation. However, we take one more step to derive formulas of much more interest. The above simple forms strongly suggest that β is a universal function of a small number of parameters. Hence, we *assume* that β is a smooth function of two parameters:

$\beta = \beta(W, x)$ for $x > x_\phi$. Then, it may be expanded as follows:

$$\beta = 1 - W - \alpha(W)x + \mathcal{O}(x^2) \quad (x_\phi < x << 1), \quad (18)$$

$$\beta = -1 + \gamma(W)x^{-1} + \mathcal{O}(x^{-2}) \quad (1 << x), \quad (19)$$

where $\alpha(W)$ and $\gamma(W)$ are positive functions of W. By integrating the first equation, we find for $L_\phi < L_0 \leq L << L_{cr}$ that

$$W(x) = \frac{W(x_0)\frac{x_0}{x}\exp\left[-\int_{x_0}^{x}\frac{\alpha(W)}{1-W}dx\right]}{1 + \left(\frac{x_0}{x}\exp\left[-\int_{x_0}^{x}\frac{\alpha(W)}{1-W}dx\right] - 1\right)W(x_0)}. \quad (20)$$

For practical purposes, the integrand may be approximated by its mean value M (> 0), and the exponential function is then reduced to $\exp[-M(x - x_0)]$. It is then predicted that as L is increased from some $L_0 \sim L_\phi$ the noise figure W first increases [due to the decrease in T in Eq. (8)], takes a maximum value at $L \sim L_{rlx}/M$, and then decreases. This interesting prediction may be experimentally confirmed by measuring the NEN in quantum wires. On the other hand, for $L_{cr} << L \leq L_0$ we find

$$W(x) = W(x_0)\frac{x_0}{x}\exp\left[-\int_{x}^{x_0}\frac{\gamma(W)}{x^2}dx\right]. \quad (21)$$

For practical purposes, $\gamma(W)$ may be approximated by a positive constant $\gamma(0)$, and then the integral becomes trivial. Equation (21) predicts an interesting deviation from the rather trivial behavior of $W \propto L^{-1}$ [3, 7] as L is decreased. This prediction may be experimentally confirmed by measuring the NEN of short conductors.

VI. NONEQUILIBRIUM NOISE OF JUNCTIONS

The above discussions have assumed a conductor of finite length L, and we have shown that W vanishes when L exceeds L_{rlx}. On the other hand, it is known that junctions, such as tunnel junctions, Schottky barriers and *pn* junctions, exhibit shot noise ($W \simeq 1$). Schottky barriers and *pn* junctions are often treated as "macroscopic", but they still exhibit shot noise. Is this consistent with our conclusion on macroscopic conductors? Let us answer this question.

In the coherent regime ($L << L_\phi, L_{rlx}$), there seems to be no essential difference between conductors and junctions. Hence, Eq. (1) holds for both. (However, most literature on tunnel junctions assumed that $T << 1$, which leads to $W = 1$.) In the macroscopic regime, however, an important difference arises. For a macroscopic conductor with $L >> L_{rlx}$, the shifted FD distribution of Eq. (9) is realized not only in the leads but also *in the conductor*. Upon the Galilean transformation to a moving frame in which $\langle J \rangle = 0$, the shifted FD distribution reduces to the equilibrium FD distribution. That is, at zero temperature, the electron system is transformed into its ground state, the Fermi sea. Since the Fermi sea exhibits no noise, the NEN is zero. For a junction, on the other hand, the bias voltage is high and the length of the junction region is relatively short, so $L >> L_{rlx}$ is not satisfied. (Recall that L_{rlx} is longer than a simple inelastic length.) Hence, the distribution of electrons *in the junction region* is very different from the shifted FD distribution, and the shot noise remains.

VII. CONCLUDING REMARKS

In the above analysis, we did not consider electron-electron (e-e) interactions. Equation (6) indicates that e-e interactions do *not* lead to $W \simeq 0$, because they cannot lead to the zero-noise distribution $\tilde{f}_1^\infty$ since the energy of the *total* electron system is conserved. That is, the energy transfer from the *total* electron system to other systems (such as phonons and photons) seems to be essential for the noise suppression. However, we do not know whether Eq. (6) is valid in the presence of e-e interactions, and future study will therefore be needed.

Note also that we have assumed one-dimensional conductors. (An exception is Eq. (21), which may be valid for any thin conductor.) Therefore, our formulas should be experimentally confirmed by measuring the noise of currents in single-mode quantum wires. For higher-dimensional or multi-mode quasi-one-dimensional conductors, our formulas may be modified. This is also a subject for future study.

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