Quantum Langevin equations for semiconductor light-emitting devices and the photon statistics at a low-injection level

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From the microscopic quantum Langevin equations (QLEs) we derive the effective semiconductor QLEs and the associated noise correlations which are valid at a low-injection level and in real devices. Applying the semiconductor QLEs to semiconductor light-emitting devices (LEDs), we obtain a formula for the Fano factor of photons that gives the photon-number statistics as a function of the pump statistics and several parameters of LEDs. Key ingredients are nonradiative processes, carrier-number dependence of the radiative and nonradiative lifetimes, and multimodeness of LEDs. The formula is applicable to the actual cases where the quantum efficiency $\eta$ differs from the differential quantum efficiency $\eta_d$, whereas previous theories implicitly assumed $\eta = \eta_d$. It is also applicable to the cases where photons in each mode of the cavity are emitted and/or detected inhomogeneously. When $\eta_d < \eta$ at a running point, in particular, our formula predicts that even a Poissonian pump can produce sub-Poissonian light. This mechanism for generation of sub-Poissonian light is completely different from those of previous theories, which assumed sub-Poissonian statistics for the current injected into the active layers of LEDs. Our results agree with recent experiments. We also discuss frequency dependence of the photon statistics.

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I. INTRODUCTION

There has been much research on quantum noise in light-emitting devices (LEDs) since the celebrated work of Shawl and Townes [1]. We here consider the quantum noise in semiconductor LEDs. This subject was first studied by Haug and Haken [2], who derived from a microscopic model useful formulas for the first- and second-order optical coherence of LEDs.

Recently, much attention has been paid to sub-Poissonian light (SPL), which has lower intensity fluctuations than the standard quantum limit [3], of various LEDs [4–14]. The mechanism for generation of SPL in LEDs is quite different from that of squeezed light in nonlinear crystals [15]. The latter mechanism is well understood as a Bogoliubov transformation of a coherent state into a squeezed state of light [16]. The former mechanism, on the other hand, is often described by the Langevin theory of lasers [3,17–21]. Previous studies on SPL in LEDs assumed that only a single mode or a few modes of photons are excited [4,6,14]. When the injection level is lowered, however, many modes of photons become relevant, and we must consider all of them. Simple theories for such a case were reported in Refs. [5,7]. However, they are too simplified so that they cannot explain recent experiments by Hirano, Kuga (HK), and co-workers [9–11], who demonstrated that the experimental results at a low-injection level (LIL) disagree with the predictions of the simplified theories. The disagreement appears when the quantum efficiency $\eta$ differs from the differential quantum efficiency $\eta_d$. HK [9] suggested that nonradiative processes might be responsible for the disagreement.

In this paper, to resolve the discrepancy between the previous theories and the experiments, we theoretically investigate the quantum noise in LEDs at a low-injection level [22].

To this end, we extend the Langevin-equation method of Chow, Koch, and Sargent [21] to treat the case where many photon modes are excited in an LED. From the microscopic quantum Langevin equations (QLEs) and the associated noise correlations, we derive the semiconductor QLEs and the noise correlations at the LIL. An important assumption is that the photon-absorption and emission rates are much smaller than the photon-escape rate from a "cavity." In the experiment [12], it has been reported that $\eta_d/\eta > 2$ in low-injection regions, whereas $\eta = \eta_d$ in high-injection regions. The difference between $\eta$ and $\eta_d$ is, in our theory, attributed to nonradiative processes and the carrier-number dependence of lifetimes. They always exist in real LEDs, and become particularly important in low-injection regions. In contrast, we can easily show that the previous simplified theories [4–7] always give $\eta = \eta_d$. Hence our theory is a minimal one to simulate real LEDs. A formula for the photon Fano factor [6] is derived. It gives the photon-number statistics as a function of the pump statistics, measuring frequency, $\eta$, $\eta_d$, and some factors arising from multimodeness of LEDs. Simplified formulas are derived for homogeneous cases, in which photons in each mode of the cavity are emitted and detected homogeneously, and for inhomogeneous cases, in which they are emitted and/or detected inhomogeneously. Using these formulas, we discuss the condition for generation of SPL in an LED. Our results agree with the experimental results.

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The validity of this assumption with the meaning of the "cavity" will be explained in the Appendix.
The paper is organized as follows. In Sec. II, we derive the semiconductor QLEs and the noise correlations at the LIL. In Sec. III, the semiconductor QLEs are used to derive a formula for the photon Fano factor. In Sec. IV, we examine the formula in various cases, and compare our theory with recent experiments [9–12]. In Sec. V, we summarize the paper.

II. DERIVATION OF QUANTUM LANGEVIN EQUATIONS AND THE NOISE CORRELATIONS AT A LOW-INJECTION LEVEL

A. Microscopic Langevin equations of an LED

Chow et al. [21] discussed the case where a single mode is excited among many-photon modes of a cavity. Since we are mainly interested in the photon statistics of LEDs at the LIL, we extend their method to treat many modes of photons. The total Hamiltonian \( \mathcal{H}_{\text{tot}} \) (which describes multimode photons in the cavity and carriers in the active layer of an LED) is written as

\[
\mathcal{H}_{\text{tot}} = \mathcal{H}_{\text{multi-ph}} + \mathcal{H}_{\text{carrier}} + \mathcal{H}_{\text{dipole}} + \mathcal{H}_{\text{many-body}} + \mathcal{H}_{\text{baths-sys}},
\]

where \( \mathcal{H}_{\text{multi-ph}} \) is the Hamiltonian of the multimode photons, \( \mathcal{H}_{\text{carrier}} \) is the annihilation operator for the photons in mode \( l \), and \( \nu_l \) is the field oscillation frequency in mode \( l \). The Hamiltonian of the electrons and holes in the active layer is \( \mathcal{H}_{\text{carrier}} \). The annihilation operators of the electron and hole of wave vector \( \mathbf{k} \) are \( c_{\mathbf{k}} \) and \( d_{-\mathbf{k}} \), respectively, and \( m_e \) and \( m_h \) are the electron and hole effective masses, respectively, and \( \varepsilon_g \) is the bare band-gap energy. The interaction among the carriers and the photons is represented by \( \mathcal{H}_{\text{dipole}} \) in the dipole approximation, with \( g_{l,\mathbf{k}}^0 \) being the bare coupling constants, and H.c. means Hermite conjugate. Note that here we consider only a direct radiative transition; however, in real devices there are other processes, and we will include them when necessary. The many-body interaction between the carriers is represented by \( \mathcal{H}_{\text{many-body}} \), the Hamiltonian of baths (or environments) by \( \mathcal{H}_{\text{baths}} \), and the interaction between the baths and the system (the carriers and photons) by \( \mathcal{H}_{\text{baths-sys}} \).

To obtain QLEs, we start by making a mean-field approximation for \( \mathcal{H}_{\text{many-body}} \), and consequently \( \varepsilon_g^0 \) and \( g_{l,\mathbf{k}}^0 \) are renormalized (see, e.g., Chap. 4 of Ref. [21]). The renormalized parameters are denoted by \( \varepsilon_g \) and \( g_{l,\mathbf{k}} \).

We then eliminate \( \mathcal{H}_{\text{baths}} + \mathcal{H}_{\text{baths-sys}} \) by using the Markov approximation [3,16–21]. As a result, the fluctuation and dissipation terms appear in the equations of motion. The generalized Einstein relation [16–18,20,21] gives the relation between the fluctuation and dissipation terms as follows:

\[
2D_{\mu} = \frac{d}{dt} \langle A_{\mu} A_{\nu} \rangle - \langle A_{\mu} \rangle \langle A_{\nu} \rangle - \langle A_{\mu} D_{\nu} \rangle - \langle A_{\mu} D_{\nu} \rangle,
\]

where \( 2D_{\mu} \) is a diffusion coefficient, \( A_{\mu} \) is a system variable, and \( D_{\mu} \) is a dissipation term in Langevin equations, i.e., \( A_{\mu} = D_{\mu} + F_{\mu} \langle P_{\mu}(t) F_{\mu}(t') \rangle \propto 2D_{\mu} \delta(t-t') \). The brackets mean the ensemble average for fluctuations.

We finally obtain the following microscopic QLEs, which describe LEDs in a microscopic scale, for the dipole operator \( \sigma_{\mathbf{k}} = d_{-\mathbf{k}} c_{\mathbf{k}} e^{i\nu_l t} \), for the electric field operator \( A_l(t) = a_l(t) e^{i\nu_l t} \), and for the electron occupation probability in \( \mathbf{k} \) space \( n_{e\mathbf{k}} = c_{\mathbf{k}}^\dagger c_{\mathbf{k}} \),

\[
\frac{d}{dt} \sigma_{\mathbf{k}} = - (\gamma + i\omega_{\mathbf{k}} - i\nu_l) \sigma_{\mathbf{k}} + i \sum_l g_{l,\mathbf{k}} A_l^\dagger (n_{e\mathbf{k}} + n_{h\mathbf{k}} - 1) + F_{\sigma_{\mathbf{k}}},
\]

\[
\frac{d}{dt} A_l = - \left( \frac{\kappa_0^l}{2} + i(\Omega_l - \nu_l) \right) A_l - i \sum_{\mathbf{k}} g_{l,\mathbf{k}}^* \sigma_{\mathbf{k}} + F_l,
\]

\[
\frac{d}{dt} n_{e\mathbf{k}} = P_{\mathbf{k}} (1 - n_{e\mathbf{k}}) - \frac{n_{e\mathbf{k}}}{\tau_{\text{nr}}} + \sum_l (ig_{l,\mathbf{k}}^* A_l^\dagger \sigma_{\mathbf{k}} + \text{H.c.}) + F_{n_{e\mathbf{k}}},
\]

where \( \gamma \) is the dipole decay (dephasing) rate, \( \hbar \omega_{\mathbf{k}} = \varepsilon_g + \hbar^2 k^2/2m_e + \hbar^2 k^2/2m_h \) is the transition energy, and \( F_{\sigma_{\mathbf{k}}} \) is the fluctuation operator for the dipole. The hole occupation probability in \( \mathbf{k} \) space is \( n_{h\mathbf{k}} = d_{-\mathbf{k}}^\dagger d_{-\mathbf{k}} \). The photon escape rate from the cavity is \( \kappa_0^l = \nu_l / Q_l \), where \( Q_l \) is the Q factor of the cavity, \( \Omega_l \) is the passive-cavity frequency, and \( F_l \) is the fluctuation operator for the electric field. The pump rate due to a current injection or optical pumping is \( P_{\mathbf{k}} (1 - n_{e\mathbf{k}}) \), where the factor \( (1 - n_{e\mathbf{k}}) \) represents the pump blocking [21]. Note that a lifetime of nonradiative decay \( \tau_{\text{nr}} \) has been introduced in Eq. (8). As discussed later, the existence of nonradiative processes is, in our model, a necessary condition for the difference between the quantum efficiency and the differential quantum efficiency to occur. The other condition is that the lifetime of radiative processes or that of nonradiative ones varies with \( n_e \). [See Eqs. (66)–(68).] Nonradiative processes might be modeled by the capture of carriers at a trapping level in an LED; however, we only need that \( \tau_{\text{nr}} \) is an implicit function of \( n_e \). These terms of pump and nonradiative decay have been phenomenologically introduced. The fluctuation operator for the electron number is \( F_{n_{e\mathbf{k}}} \).

B. Adiabatic approximation

To derive more useful forms for later discussion, we use the adiabatic approximation [19,23], and approximate the solution of Eq. (6) by

\[
\sigma_{\mathbf{k}} = \frac{i \sum_l g_{l,\mathbf{k}} A_l^\dagger (n_{e\mathbf{k}} + n_{h\mathbf{k}} - 1) + F_{\sigma_{\mathbf{k}}}}{\gamma + i\omega_{\mathbf{k}} - i\nu_l}.
\]

Substituting Eq. (9) into Eq. (7), we find...
\[ \dot{A}_I = -[\kappa_I/2 + i(\Omega_I - \nu_I)] A_I + \sum_{l'} G_{II'} A_{I'} + F_I + F_{\sigma,l}, \]

where \( G_{II'} \) is a “gain matrix”;

\[
G_{II'} = \sum_k g_{l,k}^* g_{l',k} D_{l,k}(n_{e,k} + n_{h,k} - 1),
\]

where \( D_{l,k} \) is a complex Lorentzian; \( D_{l,k} = 1/(\gamma + i(\omega_k - \nu_I)) \). A fluctuation operator \( F_{\sigma,l}(t) \) has been defined by

\[
F_{\sigma,l} = -i \sum_k g_{l,k}^* D_{l,k} F_{\sigma,k},
\]

which is associated with the coupling between the carriers and photons.

From Eq. (10), we also find the QLE for the photon-number operator \( n_I = A_I^\dagger A_I \),

\[
\frac{d}{dt} n_I = -\kappa_I n_I + \sum_{l'} [G_{II'} A_{I'} + \text{H.c.}] \\
+ [(F_{\sigma,l}^\dagger + F_{\sigma,l}^\dagger) A_I + \text{H.c.}].
\]

Substituting Eq. (9) into Eq. (8), we also find the QLE for the total electron number \( n_e = \sum_k n_{e,k} \),

\[
\frac{d}{dt} n_e = \sum_k P_{e,k}(1 - n_{e,k}) - \frac{n_e}{\tau_{nr}} - \sum_{l,l'} [G_{II'} A_{I'} + \text{H.c.}] \\
+ \sum_l F_{e,k} - \sum_{l'} [A_{l'}^\dagger F_{\sigma,l} + \text{H.c.}].
\]

C. Microscopic noise correlations of an LED

To discuss the statistical properties of light emitted from LEDs, we must determine the noise correlations. Hereafter we assume that the correlations between different modes of the photons and those between different wave numbers of carriers can be neglected [19], i.e.,

\[
\langle A_I^\dagger(t) A_I(t') \rangle = \langle n_I \rangle \delta_{t,t'},
\]

\[
\langle \sigma_{k,I}^\dagger(t) \sigma_{k',I}^\dagger(t') \rangle = \langle n_{e,k} n_{h,k} \rangle \delta_{k,k'} \delta(t-t'),
\]

\[
\langle \sigma_{k,I}(t) \sigma_{k',I}(t') \rangle = \langle (1-n_{e,k})(1-n_{h,k}) \rangle \delta_{k,k'} \delta(t-t'),
\]

\[
\langle n_{e,k}(t) n_{e,k'}(t') \rangle = \langle n_{e,k} \rangle \delta_{k,k'} \delta(t-t').
\]

We can then calculate the noise correlations, which are consistent with the QLEs (6)–(8), using Eq. (5),

\[
\langle F_{\sigma,k,I}^\dagger(t) F_{e,k,I}(t') \rangle = 2\gamma \langle n_{e,k} n_{h,k} \rangle \delta(k,k') \delta(t-t'),
\]

\[
\langle F_{\sigma,k,I}(t) F_{e,k,I}^\dagger(t') \rangle = 2\gamma \langle (1-n_{e,k})(1-n_{h,k}) \rangle \\
\times \delta(k,k') \delta(t-t'),
\]

\[
\langle F_{e,k,I}(t) F_{e,k,I}(t') \rangle = \kappa_I^0 \langle n_I \rangle \delta(t-t'),
\]

where \( \langle n_I \rangle \) is the number of thermal photons in mode \( I \) in the cavity. We have used, in Eqs. (19) and (20), the quasi-equilibrium conditions [21]

\[
2\gamma \langle n_{e,k} n_{h,k} \rangle \approx \frac{d}{dt} \langle n_{e,k} n_{h,k} \rangle,
\]

\[
2\gamma \langle (1-n_{e,k})(1-n_{h,k}) \rangle \approx \frac{d}{dt} \langle (1-n_{e,k})(1-n_{h,k}) \rangle.
\]

We can also calculate the correlations of \( F_{\sigma,l} \):

\[
\langle F_{\sigma,l,I}^\dagger(t) F_{\sigma,l,I}(t') \rangle = \langle R_{sp,I} \rangle \delta(t-t'),
\]

\[
\langle F_{\sigma,l,I}(t) F_{\sigma,l,I}^\dagger(t') \rangle = \langle R_{abs,I} \rangle \delta(t-t'),
\]

where \( R_{sp,I} \) (\( R_{abs,I} \)) denotes the spontaneous emission rate into photon mode \( I \) (the absorption rate of photon mode \( I \));

\[
R_{sp,I} = \frac{n_e}{\tau_{s,I}} = 2\gamma \sum_k |g_{l,k}|^2 \mathcal{L}_{l,k} n_{e,k} n_{h,k},
\]

\[
R_{abs,I} = 2\gamma \sum_k |g_{l,k}|^2 \mathcal{L}_{l,k}(1-n_{e,k})(1-n_{h,k}).
\]

We have used Eqs. (19) and (20), and defined \( \tau_{s,I} \) as a radiative lifetime of the carriers into mode \( I \). The (dimensionless) Lorentzian line-shape function \( \mathcal{L}_{l,k} \) is \( \mathcal{L}_{l,k} = \gamma^2/[(\gamma^2 + (\omega_k - \nu_I)^2)] \).

At this stage, we have the following QLEs:

\[
\frac{d}{dt} n_I = -\kappa_I n_I + [(F_{\sigma,l,I}^\dagger + F_{\sigma,l,I}^\dagger) A_I + \text{H.c.}],
\]

\[
\frac{d}{dt} n_e = \sum_k P_{e,k}(1 - n_{e,k}) - \frac{n_e}{\tau_{nr}} - \sum_l (R_{sp,I} - R_{abs,I}) n_I \\
+ \sum_l F_{e,k} - \sum_{l'} [A_{l'}^\dagger F_{\sigma,l,I} + \text{H.c.}],
\]

where we have used

\[
G_{II} + G_{II}^* = R_{sp,I} - R_{abs,I},
\]

and defined the renormalized photon escape rate

\[
\kappa_I = \kappa_I^0 - R_{sp,I} + R_{abs,I}.
\]
D. Semiconductor Langevin equations and the noise correlations for an LED at a low-injection level

In what follows, we assume
\[ R_{\text{sp},l} R_{\text{abs},l} \ll \kappa_l^0. \]  
(34)

From Eqs. (30) and (34), the QLE for \( n_l \) at the LIL becomes
\[ \frac{d}{dt} n_l = -\kappa_l^0 n_l + R_{\text{sp},l} + F_{n,l}, \]  
(35)

where we have defined a fluctuation operator,
\[ F_{n,l} = [(F_{\sigma,l}^+ + F_{\sigma,l}) A_l + \text{H.c.}] - [\kappa_l^0 \bar{n}(\nu_l) + R_{\text{sp},l}]. \]  
(36)

The right-hand side is averaged to be zero because the noises, \( F_{n,l} \) and \( F_{\sigma,l} \), have the Markovian property: \( \langle F_{l}^l A_l + \text{H.c.} \rangle = \kappa_l^0 \bar{n}(\nu_l) \) and \( \langle F_{\sigma,l}^l A_l + \text{H.c.} \rangle = R_{\text{sp},l} \).

From Eqs. (34) and (35), the steady-state value of the photon number becomes
\[ \langle n_l \rangle_{ss} = R_{\text{sp},l} / \kappa_l^0 \ll 1, \]  
(37)
i.e., the photon number in each mode is quite small. On the other hand, from Eqs. (31), (34), and (37), the QLE for \( n_c \) at the LIL becomes
\[ \frac{d}{dt} n_c = P - \frac{n_c}{\tau_r} - \frac{n_c}{\tau_{nr}} + F_c, \]  
(38)

where \( P = \sum_k P_{ek}(1 - n_{ek}) \) is the total pump rate, and
\[ \frac{1}{\tau_r} - \frac{\sum_l R_{\text{sp},l}}{n_c} = \sum_l \frac{1}{\tau_{lr,l}} \]  
(39)
is the radiative decay rate (\( \tau_r \) is the radiative lifetime of carriers). The fluctuation operator for the total electron number \( F_c \) is denoted by
\[ F_c = \sum_l \left[ -(A_{l}^+ F_{\sigma,l} + \text{H.c.}) + R_{\text{sp},l} \right] + \sum_k F_{ek}. \]  
(40)

Note that the lifetimes (\( \tau_{lr,l} \) and \( \tau_{r} \)) are carrier-number dependent because \( R_{\text{sp},l} \) is an implicit function of \( n_c \).

We thus obtain the final forms of the semiconductor QLEs at the LIL as follows:
\[ \frac{d}{dt} n_c = P - \frac{n_c}{\tau_r} - \frac{n_c}{\tau_{nr}} + \sigma + \Gamma_p + \Gamma_r + \Gamma_{mr}, \]  
(41)
\[ \frac{d}{dt} n_l = -\kappa_l^0 n_l + \frac{n_c}{\tau_{rl,l}} + F_{c,l} + F_{r,l}, \]  
(42)

where the noise operators \( F_{n,l}, F_{c} \) have been divided into new ones, \( \Gamma_r, F_{r,l}, \Gamma_{mr}, \Gamma_p, F_{ek,l} \). They are given by
\[ \Gamma_r = \sum_l \left[ -(A_{l}^+ F_{\sigma,l} + \text{H.c.}) + R_{sp,l} \right], \]  
(43)
\[ F_{r,l} = A_{l}^+ F_{\sigma,l} + \text{H.c.} - R_{sp,l}, \]  
(44)
\[ \Gamma_{mr} + \Gamma_p = \sum_k F_{ek}, \]  
(45)
\[ F_{ek,l} = A_{l}^+ F_{l} + F_{l}^+ A_{l}, \]  
(46)

where we have neglected the number of thermal photons, \( n(\nu_l) \), since this is the case in usual experiments. These noise operators \( (\Gamma_r, F_{r,l}, \Gamma_{mr}, \Gamma_p, F_{ek,l}) \) are associated with the radiative decay of carriers, the conversion from carriers to photons, the nonradiative decay of carriers, the (intrinsic) pump fluctuation, and the photon escape from the cavity, respectively. The mean values of the noises are defined to be zero, of course.

From above, the noise correlation for \( \Gamma_r \) can be calculated as
\[ \langle \Gamma_r(t) \Gamma_r(t') \rangle = \sum_l \left[ \langle A_{l}^+ A_{l}(t) A_{l}^+ A_{l}(t') \rangle \langle F_{\sigma,l}^+ F_{\sigma,l}(t) F_{\sigma,l}^+ F_{\sigma,l}(t') \rangle \right] + \langle A_{l}^+ A_{l}(t) A_{l}^+ A_{l}(t') \rangle \]  
\[ = \sum_l \left[ \langle R_{\text{sp},l} + R_{\text{abs},l} \rangle \langle n_l \rangle + \langle R_{\text{sp},l} \rangle \delta(t-t') \right] \]  
\[ = \sum_l \langle R_{\text{sp},l} \rangle \delta(t-t') = \frac{n_c}{\tau_r} \delta(t-t'), \]  
(47)

where, as done in Eq. (15), we have neglected, in the first and second lines, the correlations between different modes. We also have assumed the following:
\[ \Gamma_r = \sum_l \left[ -(A_{l}^+ F_{\sigma,l} + \text{H.c.}) \right], \]  
(48)

where \( A_{l}^+ \) is an operator uncorrelated with \( F_{\sigma,l} \) [24]. Thus the property of fluctuations, \( \langle \Gamma_r \rangle = 0 \), still holds. Equations (26) and (27) have also been used in the third line of Eq. (47), and Eqs. (34), (37), and (39) in the fourth line of Eq. (47).

Similarly we obtain
\[ \langle \Gamma_r(t) F_{r,l}(t') \rangle = -\frac{n_c}{\tau_{rl,l}} \delta(t-t'), \]  
(48)
\[ \langle F_{r,l}(t) F_{r,l}(t') \rangle = \frac{n_c}{\tau_{rl,l}} \delta(t-t'), \]  
(49)
\[ \langle \Gamma_{mr}(t) \Gamma_{mr}(t') \rangle = \frac{n_c}{\tau_{mr}} \delta(t-t'), \]  
(50)
\[ \langle \Gamma_p(t) \Gamma_p(t') \rangle = \langle P \rangle \delta(t-t'), \]  
(51)

where we have divided the correlation \( \langle \Gamma_{mr}(t) \Gamma_{mr}(t') \rangle + \langle \Gamma_p(t) \Gamma_p(t') \rangle \) into two parts: one is the correlation of the nonradiative processes, and the other that of the pump. These semiconductor QLEs, Eqs. (41) and (42), and the noise cor-
relations (47)–(51) were just assumed by us [22,25]. Yamanishi and Lee [5] also assumed similar ones; however, they did not include nonradiative processes.

Furthermore, we focus on the total photon flux, $N$, which can be detected at the PD surface. To this end, we must find two relations. One is the relation between the photon number in the cavity and the photon flux from the cavity. The other is the relation between the photon flux from the cavity and the photon flux detected at the PD surface. To this end, we must find nonradiative processes.

In this section, we calculate the Fano factor of photons in Refs. [22,25]. Linearizing Eqs. (41), (42), and (52) in terms of $\Delta n_e$, $\Delta n_l = n_l - (n_l)_0$, $\Delta P = P - P_0$, and $\Delta V_l = V_l - (V_l)_0$, and substituting Eqs. (55) and (57) into them, we obtain

$$\frac{d}{dt} \Delta n_e = \frac{\Delta n_e}{\tau^r} + \Gamma_r + \Gamma_m,$$

$$\frac{d}{dt} \Delta n_l = - \frac{\kappa_l^0}{\tau_{l,t}'} \Delta n_l + \frac{\Delta n_e}{\tau_{l,t}'} + F_{k,t} + F_{c,t},$$

$$\Delta V_l = \frac{\kappa_l^0}{\tau_{l,t}'} \Delta n_l - F_{k,t},$$

where we have used the following equilibrium conditions:

$$P_0 = n_{c,0} \left( \frac{1}{\tau_{0}} + \frac{1}{\tau_{m,0}} \right),$$

$$\sum_l \kappa_l^0 (n_{l,0}) = \sum_l \frac{n_{c,0}}{(\tau_{l,0})} = \sum_l V_{l0} = V_0$$

and introduced the effective lifetimes defined as

$$\tau_r' = \frac{\tau_{0}}{1 + K_r}, \quad \tau_{l,t}' = \frac{(\tau_{l,t})_0}{1 + K_{l,t}}, \quad \tau_m' = \frac{\tau_{m,0}}{1 - K_{m'}}.$$

Dropping the noise terms in Eqs. (60)–(62), and using Eq. (53), we calculate the quantum efficiency $\eta$ and the differential quantum efficiency $\eta_d$ as

$$\eta = \frac{N_0}{P_0} \frac{\beta_0 / \tau_{0}}{\beta_0 / \tau_{0} + 1 / \tau_{m,0}} = \frac{\beta_0}{1 + \epsilon_0}.$$

III. CALCULATION OF THE PHOTON FANO FACTOR

In this section, we calculate the Fano factor of photons [6] which denotes the normalized fluctuation of the photon number detected at the PD surface. Following the standard small-signal analysis used in Refs. [4,5,7,14], we expand the radiative and nonradiative lifetimes, $\tau_{r,l}[n_e]$, $\tau_l[n_e]$, and $\tau_m[n_e]$, to linear order in $\Delta n_e = n_e - n_{e,0}$.
where a total pump noise $\Delta \tilde{P}_{\text{tot}}$ has been defined as

$$\Delta \tilde{P}_{\text{tot}}(\Omega) = \Delta \tilde{P}(\Omega) + \tilde{\Gamma}_{\text{pp}}(\Omega),$$

which consists of the modulation $\Delta \tilde{P}(\Omega)$ and the intrinsic pump noise $\tilde{\Gamma}_{\text{pp}}(\Omega)$. Note that the latter noise can be suppressed by inelastic scattering in conductors [4–8,10,26].

To see the physical meaning of Eq. (70), let us introduce the Fano factor of the pump electrons (or excitons) $W_e$ and that of the photons detected at the PD surface $W_{\text{ph}}$, which are defined by

$$W_e(\Omega) = \frac{|\langle \Delta \tilde{P}_{\text{tot}}(\Omega) \rangle|^2}{P_0 T},$$

$$W_{\text{ph}}(\Omega) = \frac{|\Delta \tilde{N}(\Omega)|^2}{N_0 T},$$

where $T$ is the Fourier-integral time. We transform Eqs. (47)–(51) into the Fourier components:

$$\langle \tilde{F}_r^*(\Omega) \tilde{F}_l(\Omega) \rangle = \langle \tilde{\Gamma}_r^*(\Omega) \tilde{\Gamma}_l(\Omega) \rangle = \frac{1}{\epsilon_0} \left( \frac{\tilde{\Gamma}_r^*(\Omega) \tilde{\Gamma}_l(\Omega)}{\tilde{\Gamma}_r^*(\Omega) \tilde{\Gamma}_l(\Omega)} \right),$$

$$\langle \tilde{F}_r^*(\Omega) \tilde{F}_l(\Omega) \rangle = \frac{n \epsilon_0}{\tau_{\text{tot}}} V_0 T = \frac{P_0 T}{1 + \epsilon_0}.$$ (73)

Substituting Eqs. (70) and (73) into Eq. (54), and dividing by $N_0 T$, we finally obtain

$$W_{\text{ph}}(\Omega) = 1 - \frac{2 \eta_d \xi_1}{1 + (\Omega \tau^o)^2} + \frac{\eta_d^2}{\eta} \frac{1 + W_e(\Omega)}{1 + (\Omega \tau^o)^2} \xi_2,$$ (74)

where we have used Eqs. (53), (63), (66)–(68), and defined the following:

$$\xi_1 = \sum_{l,m} \frac{\tau_{l,m}^{\prime} \tau_{l,m}^{\prime\prime}}{(\tau_{l,m}^{\prime})^2} \xi_l \xi_m,$$

$$\xi_2 = \left[ \sum_{l} \frac{\tau_{l}^{\prime} \xi_l}{(\tau_{l}^{\prime\prime})^2} \right]^2,$$ (76)

which represent effects due to multimodeness of LEDs, and the consequences are discussed below. Note also that $\xi_1 \equiv 1, \xi_2 \equiv 1$.

Equation (74) is our main result which gives the Fano factor of the photons detected at the PD surface as a function of the Fano factor of the pump, measuring frequency, and several parameters ($\eta, \eta_d, \xi_1, \xi_2$, and $\tau^o$). Note that $1/\tau^o$ becomes a cutoff frequency of the Fano factor as well as that of the modulation [see Eq. (69)].

IV. DISCUSSION

A. Low-frequency limit

Let us examine Eq. (74) in two cases, $\Omega = 0$ and $\Omega > 0$, separately. We first discuss the low-frequency limit. This case is applicable to the most experiments, because they are usually performed at the frequency which is lower than any other relevant frequencies. Setting $\Omega \to 0$, we obtain

$$W_{\text{ph}}(0) = 1 - 2 \eta_d \xi_1 + \frac{\eta_d^2}{\eta} \frac{1 + W_e(0)}{1 + W_e(0)} \xi_2.$$ (77)
There are several cases where this expression becomes simpler.

1. The case where photons are emitted and detected homogeneously (the homogeneous case)

\( W_{\text{ph}}(0) = 1 - 2 \eta_\text{d} + \frac{\eta_\text{d}^2}{\eta} [1 + W_c(0)] \) (78)

This is the formula that we have derived elsewhere [25]. We have implicitly assumed this property (h-1) there, and this is a natural choice unless one considers a situation such that inhomogeneity due to, e.g., cavity-QED effects becomes important.

(h-2). In addition to (h-1), when the nonradiative processes do not exist and/or the carrier-number dependence of lifetimes cancels to be zero, i.e., \( r_{\text{nr}} \rightarrow \infty \) and/or \( K_r + K_{\text{nr}} = 0 \), we have \( \eta = \eta_\text{d} \) and the IL (injection-light) characteristics become straight. Hence, from Eq. (78), we obtain

\[
W_{\text{ph}}(0) = 1 - 2 \eta + \eta [1 + W_c(0)]
\]

\[
= 1 - \eta + \eta W_c(0).
\]

(79)

This is just the previous formula which is frequently used in the literature [6–9,13].

2. The case where photons are emitted and/or detected inhomogeneously (the inhomogeneous case)

Note that in the homogeneous case the factors \( \zeta_1 \) and \( \zeta_2 \) do not appear in the expressions for \( W_{\text{ph}} \) [Eqs. (78) and (79)]. On the other hand, they appear when emission and/or detection efficiencies are different among different modes. We call this general case the “inhomogeneous case.” To investigate this case, let us consider a simple case where \( K_r, t = 0 \) and \( K_{\text{nr}} \neq 0 \). In this case, \( \zeta_1 = \zeta_2 = \zeta \), where

\[
\zeta = \sum_i \frac{\tau_{0i}}{(\tau_{ri})_0 \beta_0} \xi_i \]

(80)

Hence, from Eq. (77), we obtain

\[
W_{\text{ph}}(0) = 1 - 2 \eta_\text{d} + \frac{\eta_\text{d}^2 \zeta}{\eta} [1 + W_c(0)].
\]

(81)

Compared with Eq. (78), we see that \( \eta \) and \( \eta_\text{d} \) are effectively multiplied by \( \zeta \) in Eq. (81). However, this does not mean that \( \zeta \) could be absorbed in \( \eta \) and \( \eta_\text{d} \), because they are already defined by Eqs. (66) and (67), respectively. Any redefinition would lead to disagreement with the observed IL characteristics.

3. Condition for generation of sub-Poissonian light

As another illustration of our result, we next discuss the condition for generation of sub-Poissonian light (SPL) \( (W_{\text{ph}}<1) \) with a Poissonian pump \( (W_c=1) \). For simplicity, we hereafter consider the case (h-1) [Eq. (78)], because many LEDs seem to be categorized in this case. From Eq. (78), we obtain

\[
0 < \eta_\text{d} < \eta
\]

(82)

as the condition. It can be intuitively understood that the flatter the IL characteristics, the duller the sensitivity of the LED to the pump fluctuation. Hence, if IL characteristics are like Fig. 2(b), then even a Poissonian pump \( W_c=1 \) can produce sub-Poissonian light. Note that this mechanism is completely different from those of Refs. [7,13,27], because the authors of [7,13,27] eventually make the current noise injected to the active layers below Poissonian. On the other hand, the condition (82) is equivalent to

\[
K_r + K_{\text{nr}} < 0,
\]

(83)

where we have used Eqs. (66)–(68). For simplicity, we here take \( K_{\text{nr}}=0 \), and find below what is needed for \( K_r<0 \).

The spontaneous emission (SE) rate \( n_e/\tau_r \) may be expressed approximately as \( n_e/\tau_r \propto (n_e)^p \), where \( p \) is a constant. We therefore have \( K_r \propto p-1 \) from Eq. (56). It is well known [21] that \( p \equiv 1 \) \((p=2)\) in high- (low-) injection regions for SE processes of free carriers. For exciton recombination, we have \( p\approx 1 \) at the LIL. Thus we usually have non-negative values of \( K_r \). Making use of cavity-QED effects, however, we can obtain negative values of \( K_r \). This is illustrated in Fig. 3 for a \( p \)-doped quantum well structure in a microcavity, where we have assumed the following: the conduction band is parabolic, the effective electron mass is 0.1 times the free electron mass, the doping level is high, and the cavity-QED effects prohibit SE except at the band edge.

It is seen that \( K_r \) becomes negative for the sheet carrier density \( \geq 10^{14} \text{ m}^{-2} \) when temperatures are low enough (maybe below 3 K). In this case, we obtain \( \eta_\text{d} < \eta \) [see Eqs. (66)–(68)], and even a super-Poissonian pump can produce SPL. Exciton recombination with cavity-QED effects would work better, which will be discussed elsewhere. It has been...
therefore shown that our formula \( \tau_{\text{tr}}/\tau_{\text{mr}} = 1 \) is useful to find the condition for generation of SPL, or, of course, other quantum states of light.

### B. Finite-frequency cases

Let us turn to the finite frequency cases. For simplicity, here we use Eq. (74) with property \( \tau_{\text{tr}}/\tau_{\text{mr}} = 1 \), i.e., \( \zeta_1 = \zeta_2 = 1 \), and assume the case where \( K_r = K_{nr} \) in Fig. 4. The solid lines (dashed lines) in Fig. 4 represent the cases where the nonradiative recombination is significant \( (\tau_{\text{tr}}/\tau_{\text{mr}} = 1) \) and absent \( (\tau_{\text{tr}}/\tau_{\text{mr}} = 0) \), respectively. The cutoff frequencies are indicated by vertical arrows.

1. **When \( K_r = K_{nr} = 0.5 \)**

The noiseless \( (W_e = 0) \) and Poissonian \( (W_e = 1) \) pumps are denoted by Figs. 4(a) and 4(b), respectively. When \( \tau_{\text{mr}} = \infty \) (dashed lines), we recover the results similar to those of Yamanishi and Lee [5]. We also see that the cutoff frequency (indicated by a vertical arrow) for \( \tau_{\text{mr}} \neq \infty \) becomes higher than that for \( \tau_{\text{mr}} = \infty \) [Fig. 4(a)].

2. **When \( K_r = K_{nr} = -0.5 \)**

The noiseless \( (W_e = 0) \) and Poissonian \( (W_e = 1) \) pumps are denoted by Figs. 4(c) and 4(d), respectively. We find that the existence of nonradiative processes gives smaller \( W_{ph} \) in some frequency region [Fig. 4(c)] or in the entire frequency range [Fig. 4(d)]. In particular, Fig. 4(d) shows that the Poissonian pumping can produce SPL in a wide frequency range. Figure 4(c) also shows that the cutoff frequency for \( \tau_{\text{mr}} \neq \infty \) becomes higher than that for \( \tau_{\text{mr}} = \infty \) as well as Fig. 4(a) does.

It might be difficult, however, to observe these features experimentally, because they would not appear up to \( 1/\tau_{\text{tr}} \sim 1 \) GHz. (Recently the photon Fano factor of LEDs has been measured up to 40 MHz [12].)

### C. Comparison with the experimental results

Hirano and Kuga [9] reported that the previous formula (79) disagrees with their experimental data in low-frequency regions: They measured the following ratio,

\[
\frac{W_{ph}(W_e = 0)}{W_{ph}(W_e = 1)} = r,
\]

and obtained that \( r < 1 - \eta \), whereas Eq. (79) gives \( r = 1 - \eta \). Since the measuring frequency \( \Omega_{\text{meas}} \) is low enough,
been shown that, when we observe the difference, we do not detectable. Further experimental studies are needed to explain the inequality.

When $\eta > \eta_d$ (which is usually the case in low-injection regions [9–12]), our theory gives $r < 1 - \eta$, in agreement with the experimental results. Furthermore, Hirano et al. [11] recently confirmed that Eq. (85) agrees with their experimental data (see Table I). It is seen that the values of the previous theory ($r = 1 - \eta$) are larger than the experimental values, whereas the values of Eq. (85) are closer to them. It also has been shown that, when $W_{c} = 1$, $W_{ph}$ itself is larger than 1.1. This fact also supports our formula (78) because it gives, when $W_{c} = 1$ and $\eta > \eta_d$, $W_{ph} = 1 - 2 \eta_{d} + 2 \eta_{d}^{2} / \eta > 1$, and their LEDs also have the property $\eta > \eta_d$ [9,11].

On the other hand, a simple argument [11,28] leads to

$$W_{ph}(0) = 1 - \eta + \frac{\eta_{d}^{2}}{\eta} W_{c}(0).$$

Since $|\eta - \eta_{d}|$ is small in Table I, the difference between this formula and Eq. (78) is within the experimental error and is not detectable. Further experimental studies are needed to observe the difference.

V. SUMMARY

From the microscopic QLEs (6)–(8), the associated noise correlations (19)–(23), and the assumption (34), we have derived the effective semiconductor QLEs (41), (42), and the associated noise correlations (47)–(51). The Appendix (34) is valid at a low-injection level and in real devices as explained in the Appendix. Applying the semiconductor QLEs to semiconductor LEDs, we obtain a formula (74) for the Fano factor of photons. It gives the photon-number statistics as a function of the pump statistics and several parameters of LEDs, which are defined by Eqs. (65)–(68), (75), and (76). Key ingredients are nonradiative processes, carrier-number dependence of lifetimes, and multimodeness of LEDs. The formula is applicable to the actual cases where the quantum efficiency $\eta$ differs from the differential quantum efficiency $\eta_{d}$, whereas the previous theories [4–7] turn out to give $\eta = \eta_{d}$. It is also applicable to cases where photons in each mode of the cavity are emitted and/or detected inhomogeneously. When $\eta_{d} < \eta$ at the running point, in particular, our formula predicts that even a Poissonian pump can produce super-Poissonian light (see Sec. IV A 3). This mechanism for generation of sub-Poissonian light is completely different from those of the previous theories, which assumed sub-Poissonian statistics for the current injected into the active layers of LEDs. It was shown that our results agree with recent experiments by Hirano, Kuga, and co-workers [9–12]. We have also found that, in finite frequency regions, nonradiative processes sometimes give better results (smaller $W_{ph}$ and/or a higher cutoff frequency). These will deserve further theoretical and experimental research of quantum aspects of light emitted from LEDs.

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APPENDIX: VALIDITY OF INEQUALITY (34)

We here show that inequality (34), $R_{sp,l}R_{abs,l} < k_{l}^{0}$, holds for LEDs at a low-injection level (LIL). At the LIL, pumped carriers first relax to lower-energy states which are formed by impurities, defects, spatial randomness, and so on, and then recombine to radiate photons [Fig. 5(a)]. Within the energy region of our interest, where photons are emitted, we thus have $R_{sp,l}R_{abs,l}$. In addition, in real devices, there exists a Stokes shift [Fig. 5(b)] which makes, at low temperatures, $R_{abs,l}$ even smaller. Hence we have the absorption and emission profiles such as Fig. 5(c).

We now proceed to compare $R_{sp,abs,l}$ with $k_{l}^{0}$ in Eqs. (28) and (29), $|g_{l,k}|^{2}$ is proportional to $1/V_{cavity}$ (cavity is the volume of a cavity), and $\Sigma_{k}$ is to the volume of the active layer $V_{active}$, hence we have $R_{sp,abs,l} < V_{active}/V_{cavity}$. In the
analysis of laser diodes (LDs), it is customary to take $V_{\text{active}} \approx V_{\text{cavity}}$, because LDS are made so that the lasing modes are confined in the active layer. In this case, we usually have $R_{\text{abs},l} / c \sim 10^4 \text{ cm}^{-1} \gg \kappa_l ^0 / c \sim 10^2 \text{ cm}^{-1}$ ($c$ is the velocity of light), thus we cannot obtain the inequality (34). In the case of LEDs, on the other hand, they are usually designed in such a way that the reflection coefficients on the boundary surfaces are small, hence most modes of photons of our interest are not confined in the layer. In this case, it is natural to take the “cavity” volume as big as a cube on which the detector’s surface is located [Fig. 5(d)]. Then $\kappa_l ^0$ can be estimated as $1 / \kappa_l ^0 \approx (V_{\text{cavity}})^{1/3} / c + \mathcal{Q}_{\text{device}}$, where $t_{\text{device}}$ is a time for photons to traverse the device. Since $\mathcal{Q}$ and $t_{\text{device}}$ are small for LEDs, we have $\kappa_l ^0 \approx c / (V_{\text{cavity}})^{1/3}$. Therefore, noting that $R_{\text{abs},l} \ll 1 / V_{\text{cavity}}$ and $\kappa_l ^0 \ll 1 / V_{\text{cavity}}$, we can have the relation $R_{\text{abs},l} \ll \kappa_l ^0$, if we take $V_{\text{cavity}}$ big enough. Thus we obtain inequality (34) for the case of LEDs at the LIL.

[24] See Problem 20-2 in Ref. [18].