

Quantum Zeno Effect for Exponentially Decaying Systems

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The quantum Zeno effect—suppression of decay by frequent measurements—was believed to occur only when the response of the detector is so quick that the initial tiny deviation from the exponential decay law is detectable. However, we show that it can occur even for exactly exponentially decaying systems, for which this condition is never satisfied, by considering a realistic case where the detector has a finite energy band of detection. The conventional theories correspond to the limit of an infinite bandwidth. This implies that the Zeno effect occurs more widely than expected thus far.

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The survival probability $s(t)$ of an unstable quantum system generally decreases quadratically with time t at the beginning of the decay, and later follows the well-known exponentially decay law. The crossover takes place at $t \sim \tau_j$, which is called the jump time. Noticing this fact and simply applying the projection postulate on measurements, it was predicted that frequent measurements with time intervals τ result in the suppression of the decay if τ is small enough to satisfy $\tau \lesssim \tau_j$ [1]. Similarly, the suppression was supposed to occur by continuous measurement using an apparatus with short response time τ satisfying $\tau \lesssim \tau_j$ [2]. This prediction, called the quantum Zeno effect (QZE), is one of the most curious results of the quantum measurement, and has been attracting much attention [1–13]. However, in most unstable systems, τ_j is extremely small so that the condition $\tau \lesssim \tau_j$ cannot be satisfied by existing detectors. It has been therefore believed that the QZE would not occur in most unstable systems. However, these conventional arguments on the QZE need to be reconsidered, because, in general, real measurement processes are not “projective measurements” that are described by the projection postulate [13,14]. A more general and accurate way of analyzing measurement processes is to apply the laws of quantum theory to a larger system which include *both* the original quantum system to be measured *and* a part of the measuring apparatus [13,14]. Such an approach has been successfully applied, e.g., to quantum optics [14–17] and the QZE [2,7]. This enables one to study not only the temporal evolution of the unstable system but also the response of the measuring apparatus. Moreover, one can discuss general measurement processes such as measurement with a finite probability of error [7,17].

In this paper, we apply this modern approach to the QZE in a realistic situation where the unstable system is monitored continuously using a measuring apparatus

(detector) with a finite energy band of detection. Furthermore, we consider the case of “indirect measurement” [7], in which the detector acts only on the decay products, because some of the previous works assuming direct interactions have often been criticized as not being a genuine QZE. We show that the QZE can take place even in systems that exactly follow the exponential decay law, for which τ_j is infinitesimal, hence, the conventional condition for the QZE, $\tau \lesssim \tau_j$, is never satisfied. We clarify its physical origin, derive the conditions for inducing the QZE in such systems, and show that the conventional projection-based theory can be reproduced in the ideal, but unrealistic limit of an infinite detection bandwidth. Our results imply that the response time required for the QZE is not so short, the required jump time is not so long, and, hence, the QZE may take place much more widely than expected thus far.

As the unstable quantum system, we consider the excited state of a two-level atom, which decays to the ground state with a finite lifetime accompanying emission of a photon. By detecting the emitted photon, the observer can know the decay of the atom. As the photon detection process, we consider a standard one [14–16]: In the photo-detector, the emitted photon is absorbed by a semiconductor and an electron-hole pair is generated. It finally yields a macroscopic signal after magnification processes, by which the observer knows the decay of the excited atom. Following the quantum theory of photon counting [14–16], we treat the relevant part of the detector, i.e., electron-hole pairs, as a part of the total quantum system. The electron-hole pairs can be described as bosonic elementary excitations, because their density is low in the detection process. The total quantum system is therefore composed of three parts: (i) a two-level atom with the ground state $|g\rangle$ and the excited state $|x\rangle$ with the transition energy Ω (we hereafter take $\hbar = c = 1$); (ii) the

photon field, whose eigenmodes are labeled by the wave vector \mathbf{k} ; and (iii) the field of the electron-hole bosonic excitation in the detector. By taking the energy of $|g\rangle$ as the origin of the energy, the Hamiltonian of the total system is given as follows:

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_1 + \mathcal{H}_2, \quad (1)$$

$$\mathcal{H}_0 = \Omega|x\rangle\langle x|, \quad (2)$$

$$\mathcal{H}_1 = \int d\mathbf{k} [(\xi_{\mathbf{k}}|x\rangle\langle g|b_{\mathbf{k}} + \text{H.c.}) + kb_{\mathbf{k}}^\dagger b_{\mathbf{k}}], \quad (3)$$

$$\mathcal{H}_2 = \iint d\mathbf{k} d\omega [(\sqrt{\eta_{\mathbf{k}}}b_{\mathbf{k}}^\dagger c_{\mathbf{k}\omega} + \text{H.c.}) + \omega c_{\mathbf{k}\omega}^\dagger c_{\mathbf{k}\omega}]. \quad (4)$$

Here, \mathcal{H}_1 represents the atom-photon interaction, where $b_{\mathbf{k}}$ is the annihilation operator for the photon with wave vector \mathbf{k} , whose energy is $k = |\mathbf{k}|$. The atom-photon part $\mathcal{H}_0 + \mathcal{H}_1$ constitutes the original unstable quantum system to be measured. \mathcal{H}_2 represents the interaction between the emitted photon and the detector. Every photon mode is coupled to a continuum of the bosonic elementary excitations in the detector, whose annihilation operator is denoted by $c_{\mathbf{k}\omega}$. The commutation relations are orthonormalized as $[b_{\mathbf{k}}, b_{\mathbf{k}'}^\dagger] = \delta(\mathbf{k} - \mathbf{k}')$ and $[c_{\mathbf{k}\omega}, c_{\mathbf{k}'\omega'}^\dagger] = \delta(\mathbf{k} - \mathbf{k}')\delta(\omega - \omega')$.

In order to demonstrate that the QZE does occur even when $\tau \geq \tau_j$, for which the QZE cannot be expected according to the conventional theories, we study the limiting case of infinitesimal τ_j [18]. Namely, we investigate the case where the survival probability $s(t)$ of the initial excited state exactly follows the exponential decay law, $s(t) = \exp(-\gamma t)$, when the original system is not measured, i.e., when $\mathcal{H}_2 = 0$. In the present model, we obtain such an exact exponential decay for the original quantum system (described by $\mathcal{H}_0 + \mathcal{H}_1$) by putting the atom-photon coupling after angular integration to be independent of the photon energy k :

$$\int d\mathbf{q} |\xi_{\mathbf{q}}|^2 \delta(|\mathbf{q}| - k) \equiv \frac{\gamma}{2\pi}, \quad (5)$$

and by extending the lower limit of the photon energy to $-\infty$. In this case, τ_j becomes infinitesimal, and $\tau > \tau_j$ for any finite τ . We show in the following that the QZE can occur even in this case [18]. The point is that any real detector has a finite bandwidth of detection. In contrast, the use of the projection hypothesis corresponds to the use of an ideal but unrealistic detector that has an infinite bandwidth.

To demonstrate this, we here take the following simple form for $\eta_{\mathbf{k}}$:

$$\eta_{\mathbf{k}} = \eta_{\mathbf{k}} \equiv \frac{\eta/2\pi}{1 + [(k - \Omega)/\Delta]^n}, \quad (6)$$

where n is a large even integer (we take $n = 6$ in the numerical examples). In this case, photons within the energy range $|k - \Omega| \leq \Delta$ are counted by the detector with a time scale of $\eta^{-1} \equiv \tau$, while photons outside of this energy range are not counted. Thus, the detector has a detection band centered around the atomic transition energy Ω with (half) the bandwidth $\approx \Delta$. The atomic decay is continuously monitored with this detector.

To see the temporal evolution of the whole system, we put $|\psi(t)\rangle = e^{-i\mathcal{H}t}|x, 0, 0\rangle = f(t)|x, 0, 0\rangle + \int d\mathbf{k} f_{\mathbf{k}}(t) \times |g, \mathbf{k}, 0\rangle + \iint d\mathbf{k} d\omega f_{\mathbf{k}\omega}(t)|g, 0, \mathbf{k}\omega\rangle$, and define three probabilities of physical interest: $s(t) = |f(t)|^2$ (survival probability of the atom), $\varepsilon(t) = \int d\mathbf{k} |f_{\mathbf{k}}(t)|^2$ (probability that the atom has decayed but the emitted photon is not absorbed by the detector), and $r(t) = \iint d\mathbf{k} d\omega |f_{\mathbf{k}\omega}(t)|^2$ (probability that the emitted photon is absorbed). $r(t)$ can be interpreted as the probability of getting a detector response, whereas $\varepsilon(t)$ is the probability that the detector reports an erroneous result. One of the advantages of the present theory is that all of these interesting quantities can be calculated. A numerical example is shown in Fig. 1. Since $\Delta \gg \gamma$ in this example, the atomic natural linewidth ($\sim \gamma$) is completely covered by the detection band ($\sim \Delta$), and the emitted photon is counted almost perfectly [$r(t) \approx 1$ for $t \rightarrow \infty$]. It is observed that $r(t)$ follows the decay probability $1 - s(t)$ with a delay time $\approx \tau (= \eta^{-1})$. Hence, τ is the response time of the detector. To be more precise, τ is the *lower limit* of the response time because additional delays in the response, such as delays in signal magnification processes, may occur in practical experiments. In discussing fundamental physics, the limiting value is more significant than practical values, which depend strongly on detailed experimental conditions. A typical value of τ for GaAs is 10^{-15} s, which is much shorter than the practical response times of commercial photodetectors, which range from 10^{-6} to 10^{-13} s.

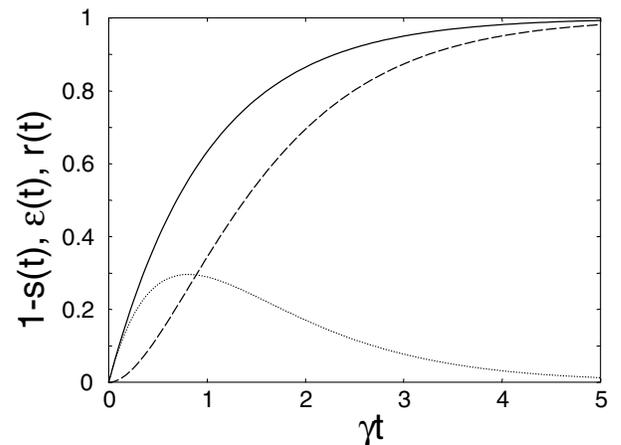


FIG. 1. Temporal evolution of $1 - s(t)$ (solid line), $\varepsilon(t)$ (dotted line), and $r(t)$ (dashed line). The parameters are chosen as $2\pi\Delta/\gamma = 100$ and $\eta/\gamma = 1.5$. It is seen that $r(t)$ follows $1 - s(t)$ with a delay time about $\tau \equiv \eta^{-1}$.

In order to see the QZE, we investigate how the survival probability $s(t)$ is affected by the detector parameters such as the response time τ and the detection bandwidth Δ of the detector. For this purpose, we transform \mathcal{H} into the following renormalized form [7]:

$$\mathcal{H} = \mathcal{H}_0 + \bar{\mathcal{H}}_1 + \bar{\mathcal{H}}_2, \quad (7)$$

$$\bar{\mathcal{H}}_1 = \int d\mu [(g_\mu |x\rangle\langle g| B_\mu + \text{H.c.}) + \mu B_\mu^\dagger B_\mu], \quad (8)$$

where $\bar{\mathcal{H}}_2$ is composed of terms which are decoupled from the atom. Here, B_μ is a coupled-mode operator that is a linear combination of b_k and $c_{k\omega}$ [7], which is orthonormalized in terms of a one-dimensional label μ as $[B_\mu, B_{\mu'}^\dagger] = \delta(\mu - \mu')$, and $|g_\mu|^2$ is given by

$$|g_\mu|^2 = \frac{\gamma}{2\pi} \int dk |g_{\mu,k}|^2, \quad (9)$$

where $|g_{\mu,k}|^2 = \eta_k / |\mu - k - i\pi\eta_k|^2$. Thus, the atom is coupled to a single continuum of B_μ with the form factor $|g_\mu|^2$, which we hereafter call the renormalized form factor. $|g_{\mu,k}|^2$ represents the contribution to $|g_\mu|^2$ from photons with energy k . It is broadened to have a finite width η_k by the measurement, satisfying a sum rule, $\int d\mu |g_{\mu,k}|^2 = 1$.

When the atom-photon system is free from the measurement, i.e., when $\eta = 0$, the form factor reduces to the “free” value, $|g_\mu|^2 = \gamma/2\pi$, and the atomic decay exactly follows the exponential law, $s(t) = \exp(-\gamma t)$. When the detector is present, on the other hand, $|g_\mu|^2$ is modified (renormalized) as shown in Fig. 2, where $|g_\mu|^2$ is plotted for several values of η/Δ . It is seen that $|g_\mu|^2$ is decreased for $|\mu - \Omega| \lesssim \Delta$ (inside the detection band) and is increased for $|\mu - \Omega| \gtrsim \Delta$ (outside). For larger $|\mu - \Omega|$, modification of $|g_\mu|^2$ becomes small, approaching zero (i.e., $|g_\mu|^2 \rightarrow \gamma/2\pi$). The figure also shows that

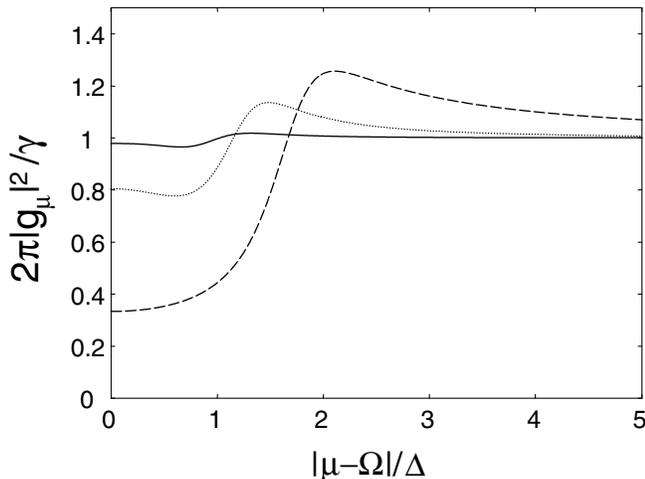


FIG. 2. Plot of $|g_\mu|^2$. $\eta/2\pi\Delta$ is chosen at 0.01 (solid line), 0.1 (dotted line), and 1 (dashed line).

the modification of $|g_\mu|^2$ occurs more significantly for larger η/Δ . This can be seen clearly in the limit of $n \rightarrow \infty$, where $|g_\mu|^2$ can be analytically evaluated and its value at the atomic transition energy is given by $|g_\Omega|^2 = (\gamma/\pi^2) \arctan(2\Delta/\eta)$. Thus, $|g_\Omega|^2$ becomes significantly smaller than the free value $\gamma/2\pi$ when $\eta/\Delta \gtrsim 1$, i.e., when

$$\tau \cdot \Delta \lesssim 1. \quad (10)$$

The survival probability $s(t)$ is governed by the functional form of $|g_\mu|^2$. In order to visualize the decay rate, $\ln s(t)/(\gamma t)$ is plotted in Fig. 3. The figure clarifies the following two-stage behavior of the atomic decay: In the first stage ($t \lesssim \Delta^{-1}$), the atom decays with the free decay rate γ , while, in the second stage ($t \gtrsim \Delta^{-1}$), the atom decays with a suppressed decay rate $2\pi|g_\Omega|^2$. For the values of parameters chosen in Fig. 3, the atom is kept almost undecayed [$s(t) \simeq 1$] in the first stage, and significant decay occurs in the second stage with the suppressed decay rate. Namely, the QZE surely takes place, although the system exhibits an exact exponential decay in the absence of measurement.

The two-stage behavior can be understood with a help of the perturbation theory. Applying the lowest-order perturbation to the renormalized form Eq. (7), we obtain the decay probability as

$$1 - s(t) = \int d\mu |g_\mu|^2 \frac{\sin^2[(\mu - \Omega)t/2]}{[(\mu - \Omega)/2]^2}. \quad (11)$$

Taking into account that the main contribution in the integral comes from the region of μ satisfying $|\mu - \Omega| \lesssim 2\pi\tau^{-1}$, we evaluate the right-hand side in two cases: In the case of $t \ll \Delta^{-1}$, $|g_\mu|^2$ can be approximated by $|g_\infty|^2 = \gamma/2\pi$, which gives the free decay rate

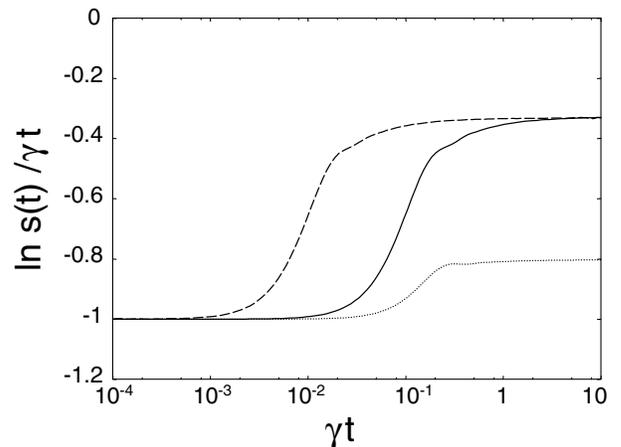


FIG. 3. Plot of $\ln s(t)/(\gamma t)$, i.e., the time dependence of the decay rate. $\{2\pi\Delta/\gamma, \eta/\gamma\}$ are chosen at $\{100, 100\}$ (solid line), $\{100, 10\}$ (dotted line), and $\{1000, 1000\}$ (broken line). The decay rate changes from the free rate γ to the suppressed rate $2\pi|g_\Omega|^2$ at $t \sim \Delta^{-1}$.

$1 - s(t) = \gamma t$; whereas in the opposite case of $t \gg \Delta^{-1}$, $|g_{\mu}|^2$ can be approximated by $|g_{\Omega}|^2$, which gives the suppressed decay rate $1 - s(t) = 2\pi|g_{\Omega}|^2 t$. Thus, the decay rate changes from the free rate to the suppressed rate at $t \sim \Delta^{-1}$.

The QZE occurs for exponentially decaying systems when the following two conditions are satisfied: (i) The transition from the first to the second stage should occur before the atom decays. Since the survival probability at $t \sim \Delta^{-1}$ is given by $s(\Delta^{-1}) \approx \exp(-\gamma/\Delta)$, this condition is expressed as

$$\gamma/\Delta \ll 1, \quad (12)$$

which means that the detection band should completely cover the natural linewidth of the atom. (ii) The decay rate in the second stage ($t \gtrsim \Delta^{-1}$) should be significantly suppressed from the free decay rate. This condition is expressed by inequality (10).

The latter condition explains why the QZE was not obtained for exponentially decaying systems by theories that assumed projective measurement. By applying the projection operator, the quantum coherences between $|x, 0\rangle$ and $|g, k\rangle$ are destroyed regardless of the energy of the emitted photon. Therefore, the projection-based theory corresponds to $\Delta \rightarrow \infty$ [19]. In such a limit, however, inequality (10) cannot be satisfied and the QZE never occurs. Since Δ of any real detector is finite, such a limit is rather unphysical. It might sound strange that a ‘‘better’’ detector with larger Δ cannot induce the QZE. However, for detecting photons whose energy k lies in the natural linewidth of the atom, $|k - \Omega| \leq \gamma$, such a detector is not better (nor worse) than a detector with smaller Δ , because the detection efficiencies of both detectors are almost 100% if $\Delta \gtrsim \gamma$ is satisfied. In the $\Delta \rightarrow \infty$ limit, the effect of individual broadenings of $|g_{\mu,k}|^2$ is perfectly smeared out in $|g_{\mu}|^2$ after the k integration in Eq. (9).

In conclusion, we make three remarks. First, we have assumed continuous measurement. Regarding frequent discrete measurements, we note that real detectors have finite response times τ , although the conventional theories on the QZE often assumed that each measurement was an instantaneous projective measurement. In some cases, continuous measurement with the response time τ corresponds to frequent instantaneous measurements with the intervals of order τ [2]. However, this does *not* mean that continuous measurement *always* corresponds to some frequent instantaneous projective measurements. In fact, the result of this work is a counterexample. Second, we remark that the QZE is closely related to the cavity quantum electrodynamics (QED) [20]. In fact, Eqs. (1)–(4) are similar to those describing the cavity QED, in which b_k correspond to discretized cavity modes while $c_{k\omega}$ correspond to outer modes, or, when the cavity is composed of photoabsorptive materials, to ex-

citations in the material. In principle, these quanta can be utilized for detection of the decay, if appropriate magnification processes are followed. In such cases, the lower limits of the response time τ would be determined in terms of the material parameters such as η_k . The present theory not only demonstrates the QZE for the exponentially decaying systems, but also reveals that the cavity QED is related to the QZE through such implicit relations [21]. Finally, we have studied in this paper the case where the decay is suppressed. The possibility of acceleration of the decay by measurement [5–7] in exponentially decaying systems will be discussed elsewhere.

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