Universal Properties of Response Functions of Nonequilibrium States

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References:

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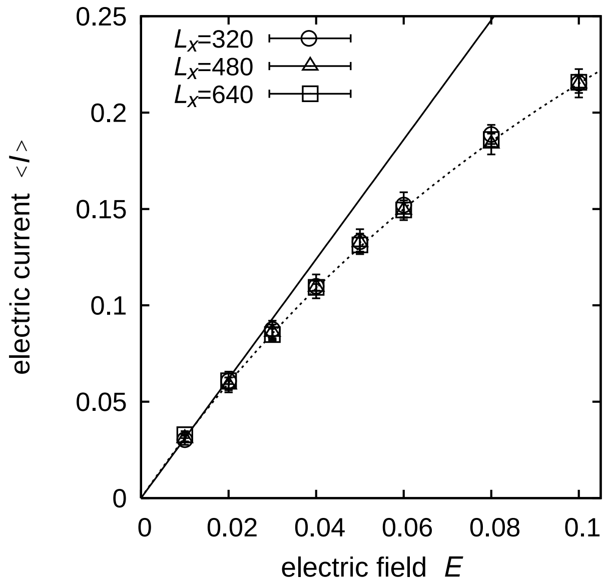
Two regimes of nonequilibrium statistical mechanics

1. **Linear nonequilibrium regime**

   - Close to equilibrium, response is linear
   - Kubo formula $\longrightarrow$ many universal properties

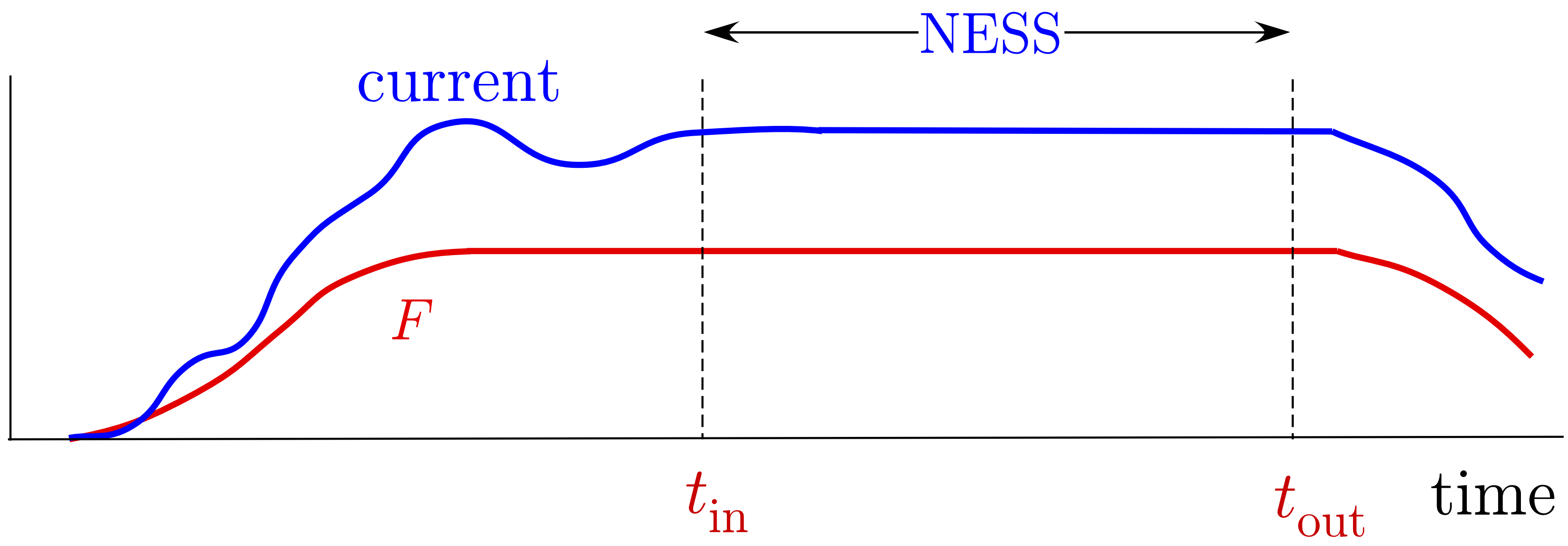
2. **Nonlinear nonequilibrium regime** $\longleftarrow$ This talk

   - Far from equilibrium, response is nonlinear.
   - Kubo formula breaks down $\longrightarrow$ any universal properties?



Response function of a nonequilibrium state

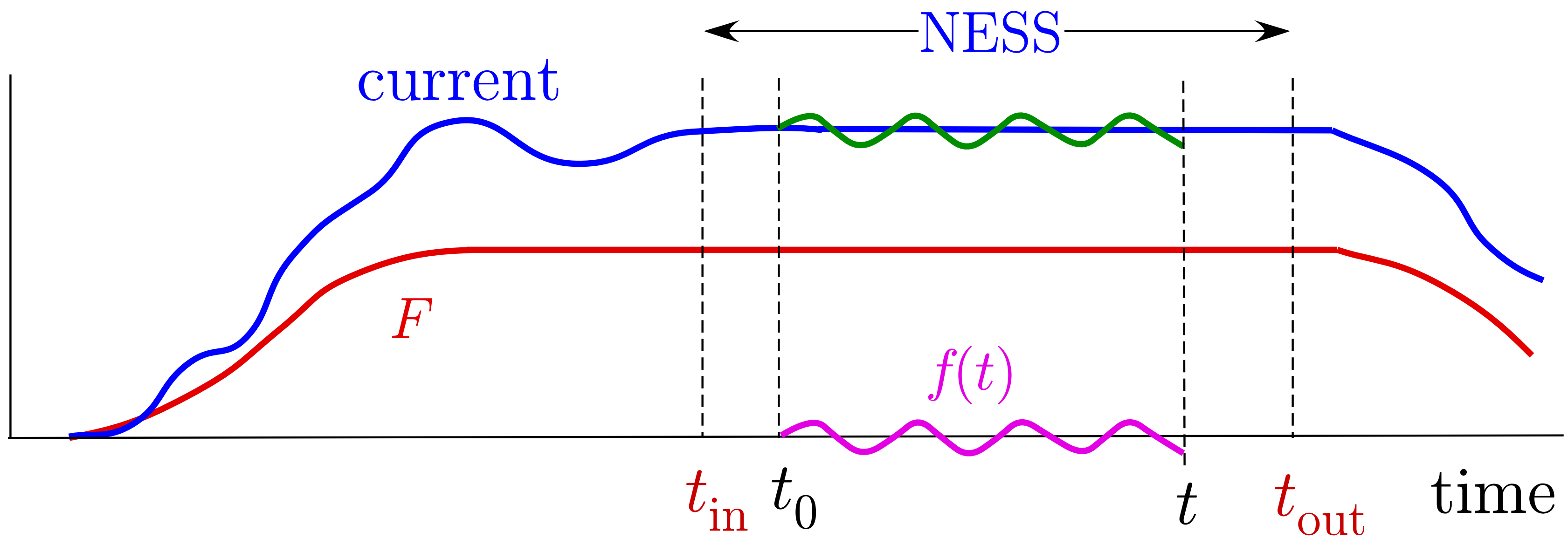
- Apply a strong static field F (pump field).



A nonequilibrium steady states (NESS) is realized for $[t_{\rm in}, t_{\rm out}]$.

- Further apply a weak and time-dependent probe field $f(t)$ for $t \geq t_0$,

$$F(t) = F + f(t).$$



- Response of the NESS to $f(t)$: see the response,

$$\Delta A(t) \equiv \langle A \rangle_{F+f}^{t} - \langle A \rangle_F,$$

of a macroscopic variable A (such as $\vec{S} = \sum_{\boldsymbol{r}} \vec{s}(\boldsymbol{r})$).

- To the linear order in f,

$$\Delta A(t) = \int_{t_0}^{t} \Phi_F(t - t') f(t') dt'$$

should hold.

Q. Universal properties of Φ_F?

An Example of Universal Properties

Let

$$\Xi_F(\omega) \equiv \int_0^\infty \Phi_F(\tau) e^{i\omega\tau} d\tau.$$

then the following **sum rule** holds;

$$\int_{-\infty}^{\infty} \mathrm{Re}\, \Xi_F^{AB}(\omega) \frac{d\omega}{\pi} = \left\langle \frac{1}{i\hbar} \left[\hat{B}, \hat{A}\right] \right\rangle_F ,$$

where

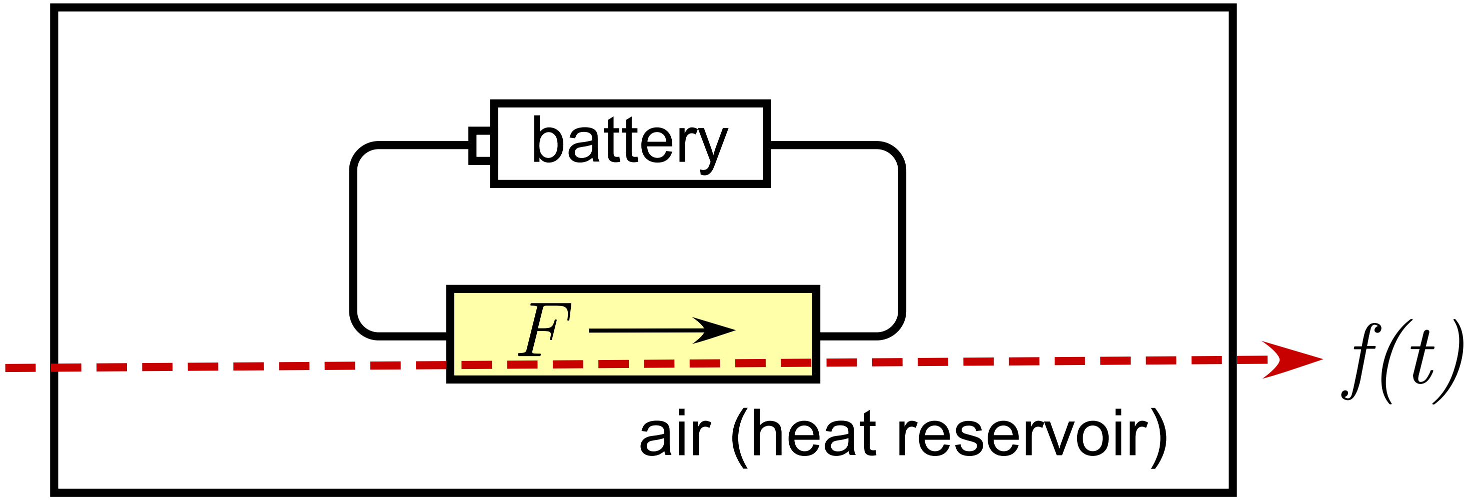
$\hat{A}$: observable of interest

$\hat{B}$: observable that couples to $f(t)$ via the interaction term, $-\hat{B} f(t)$

$\langle \cdot \rangle_F \equiv \mathrm{Tr}\,(\hat{\rho}_F \,\cdot\,)$: expectation value in the NESS $\left(\hat{\rho}_F \equiv \mathrm{Tr}' \left[\hat{\rho}_F^{\mathrm{tot}}(t)\right]\right)$

This relation is general and universal!

Example: electrical conductor



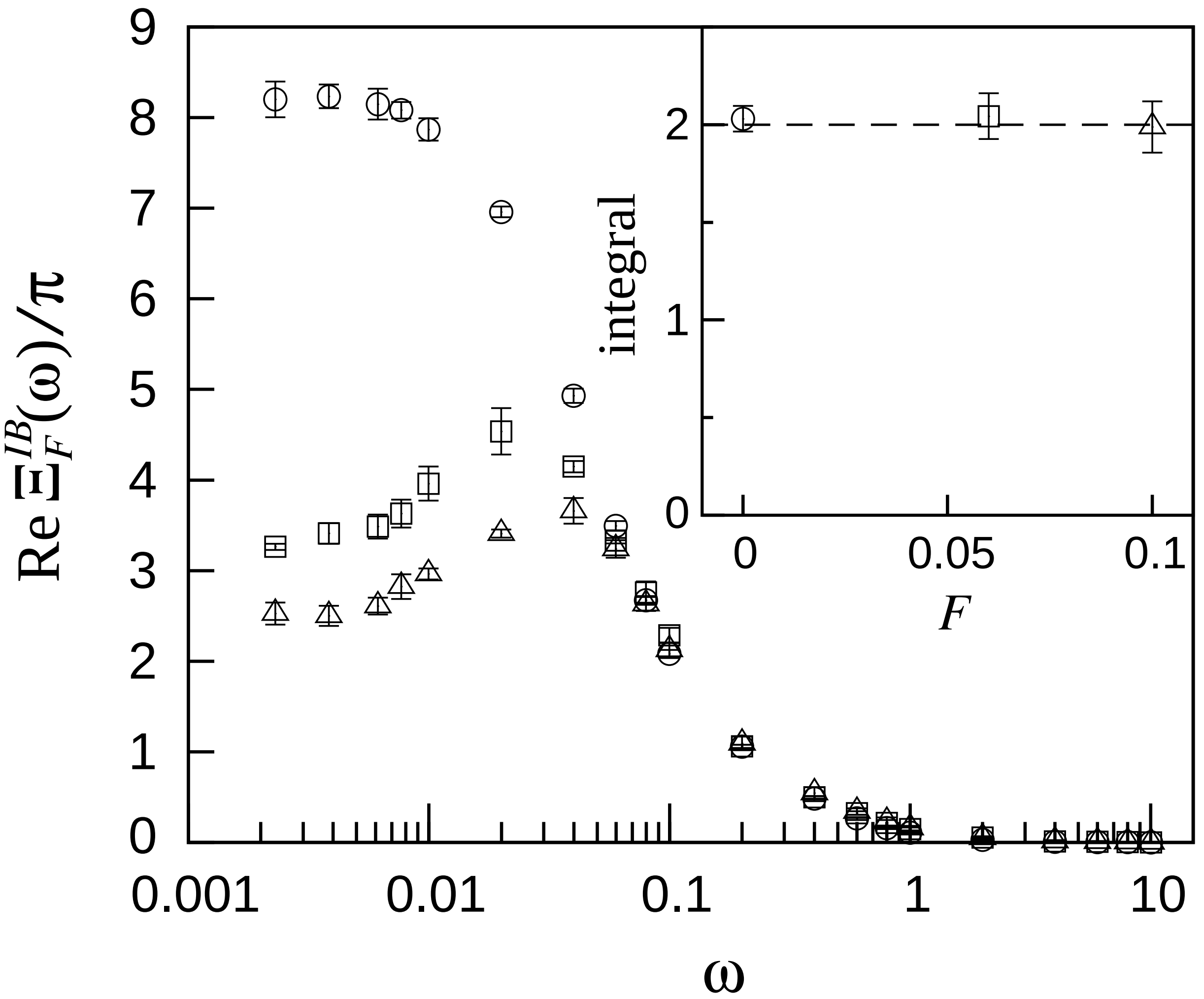
When $A = I$ (electric current averaged over the x direction),

$$\Delta I(t) \equiv \langle I \rangle_{F+f}^{t} - \langle I \rangle_F$$
$$= \int_{t_0}^{t} \Phi_F(t-t') f(t') dt' + o(f).$$

The sum rule says

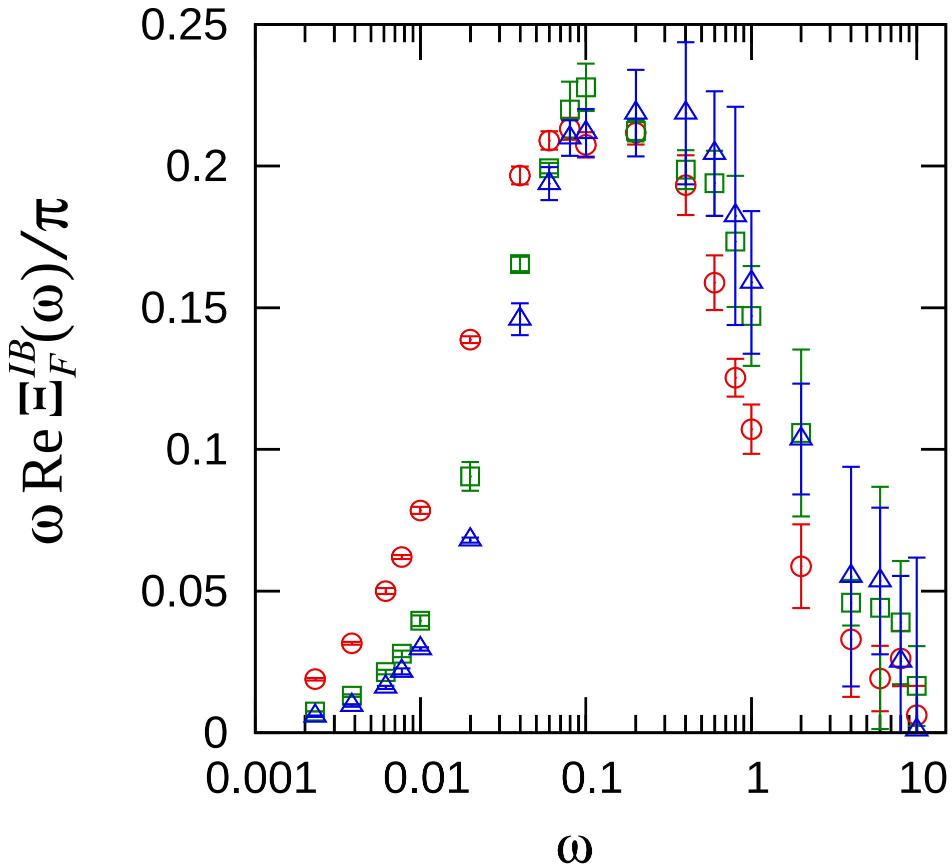
$$\int_{-\infty}^{\infty} \mathrm{Re}\, \Xi_F(\omega) \frac{d\omega}{\pi} = \frac{e^2 N_e}{mL} \quad : \text{ independent of } F\,!$$

$$\textbf{Sum rule:} \quad \int_{-\infty}^{\infty} \mathrm{Re}\, \Xi_F^{IB}(\omega) \frac{d\omega}{\pi} = \frac{e^2 N_e}{mL} \ : \ \text{indepdendent of } F$$



$F = 0$ (circles), 0.06 (squares) and 0.1 (triangles).

$$\int_{-\infty}^{\infty} \mathrm{Re}\,\Xi_F^{IB}(\omega)\frac{d\omega}{\pi} = 2\int_{-\infty}^{\infty} \omega \mathrm{Re}\,\Xi_F^{IB}(\omega)\frac{d\ln\omega}{\pi} = \frac{e^2 N_e}{mL}$$



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