A Brief Review of Quantum Information Aspects of Black Hole Evaporation

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Reference:

M. Hotta and A. Sugita, Prog. Theor. Exp. Phys, 123B04 (2015).

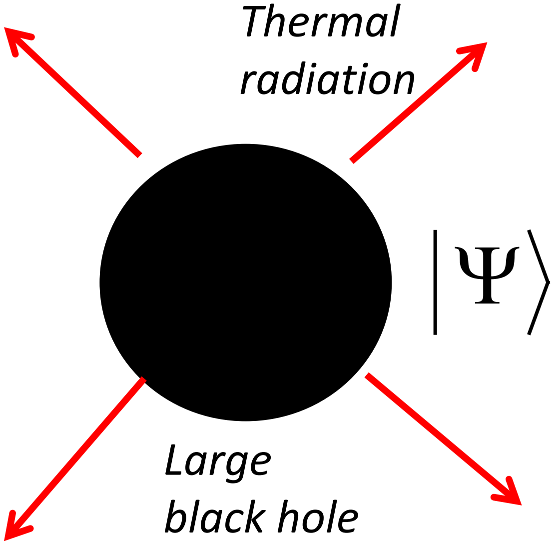
M. Hotta, R. Schützhold and W. G. Unruh, Phys. Rev. D 91, 124060 (2015).

M. Hotta, Y. Nambu, and K. Yamaguchi, arXiv:1706.07520.

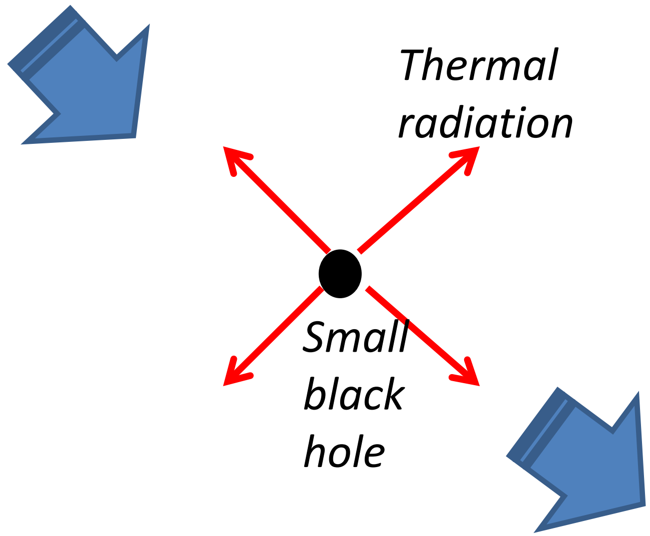
+ ….

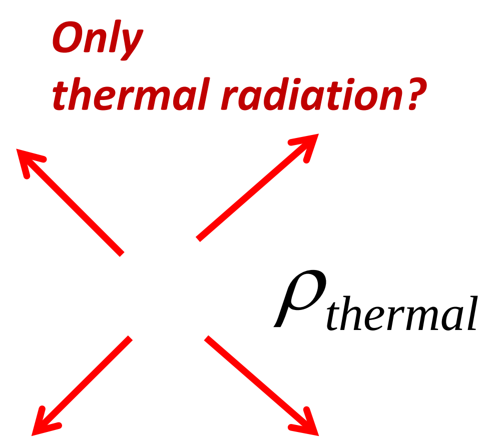
The Information Loss Problem

Hawking (1976)



$$\hat{U}|\Psi\rangle\langle\Psi|\hat{U}^{\dagger} \neq \rho_{thermal}$$

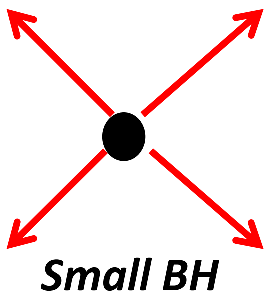




Unitarity breaking?

Information is lost!?

Why is the information loss problem so serious?



Too small energy to leak the huge amount of information.

(Aharonov, et al 1987; Preskill 1992.)

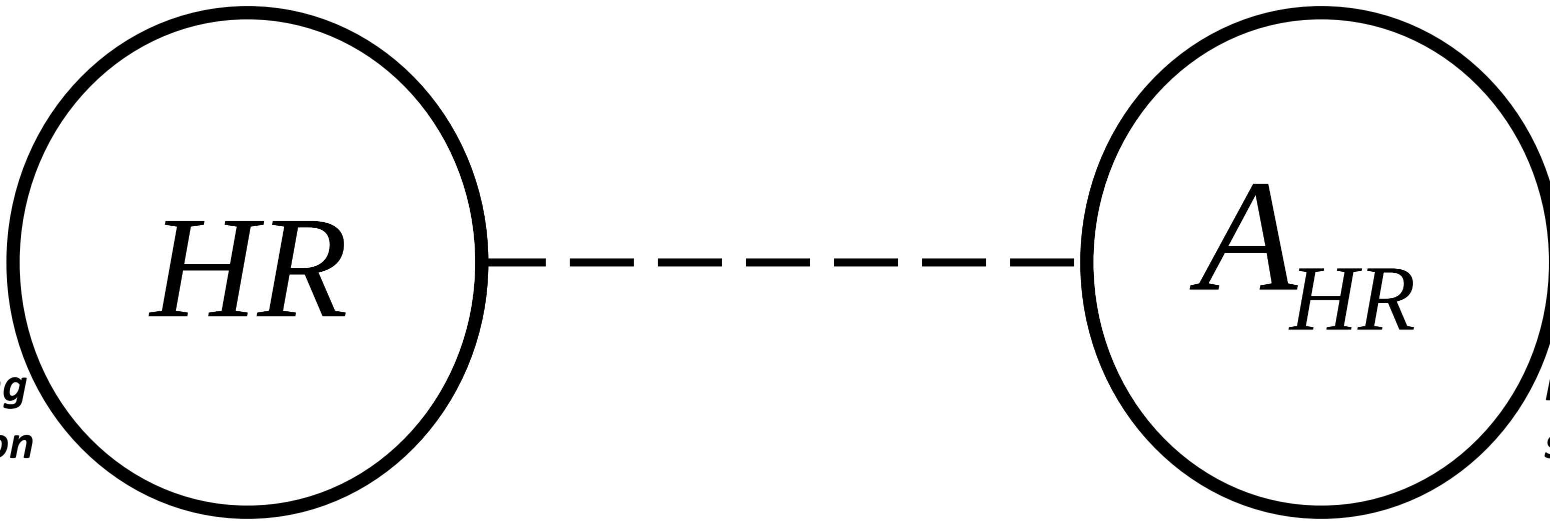
If the horizon prevents enormous amount of information from leaking until the last burst of BH, only very small amount of BH energy remains, which is not expected to excite carriers of the information and spread it out over the outer space.

Purification Problem of Hawking Radiation:
from a modern viewpoint of information loss

$$\rho_{HR} = \sum_n p_n |n\rangle_{HR} \langle n|_{HR}$$

Mixed state

Hawking radiation system



Partner system

$$|\Psi\rangle_{HRA_{HR}} = \sum_n \sqrt{p_n} |n\rangle_{HR} |u_n\rangle_{A_{HR}}$$

Composite system in a pure state

What is the final purification partner of the Hawking radiation?

(1) Nothing, Information Loss

(2) Exotic Remnant (Aharonov, Banks, Giddings,…)

(3) Baby Universe (Dyson,..)

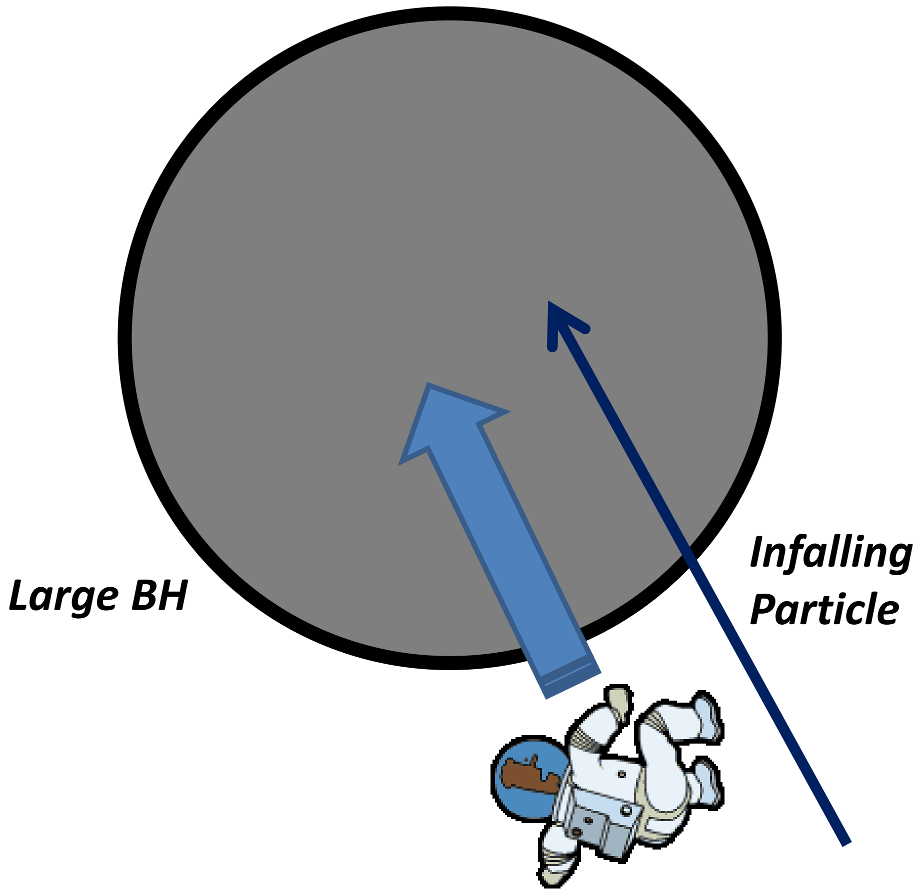
(4) Radiation Itself (Page,…)

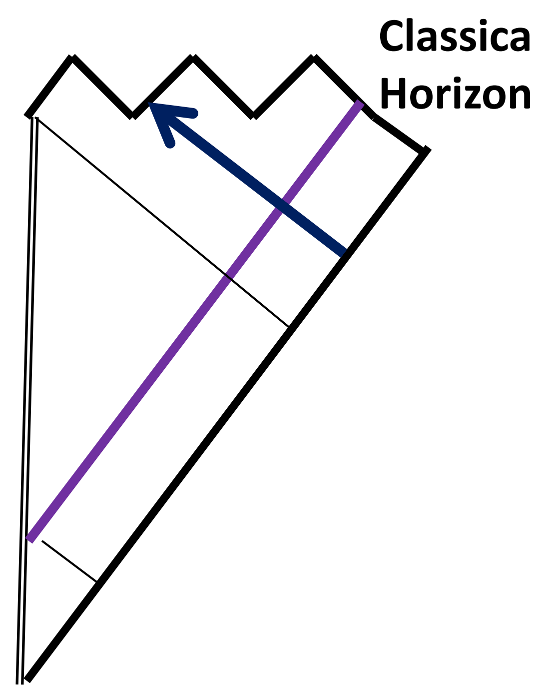
- *Black Hole Complementarity (‘t Hooft, Susskind, …)*
- *Fuzzi ball, **Firewall** (Mathur, Braunstein, AMPS, …)*

(4) Radiation

○ Black Hole Complementarity

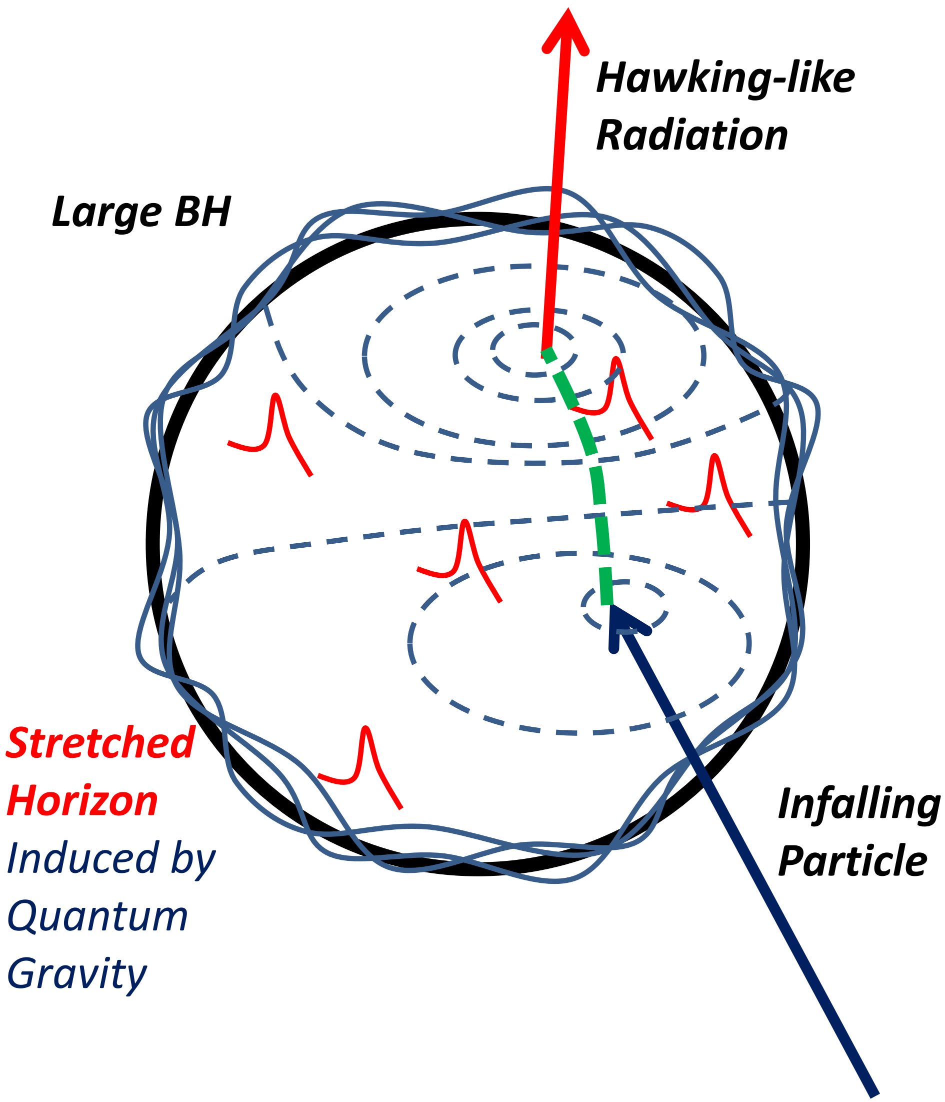
From the viewpoint of free-fall observers, no drama happens across the horizon.





(4) Radiation

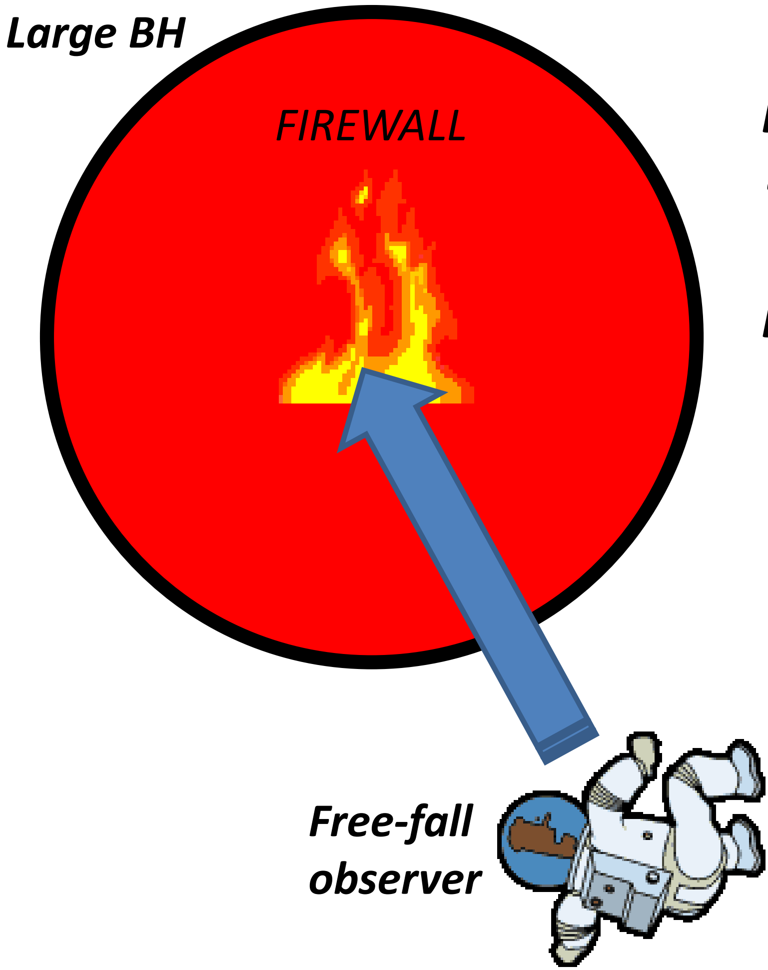
○ Black Hole Complementarity



From the viewpoint of outside observers, the stretched horizon absorbs and emits quantum information so as to maintain the unitarity.

(4) Radiation

○ Firewall



A FIREWALL on the horizon burns out free-fall observers. The inside region of BH does not exist!
This is argued from monogamy of entanglement.

This scenario is criticized in this talk.

What is the final purification partner of the Hawking radiation?

(1) Nothing, Information Loss

(2) Exotic Remnant (Aharonov, Banks, Giddings,...)

(3) Baby Universe (Dyson,..)

(4) Radiation Itself (Page,...)

- *Black Hole Complementarity ('t Hooft, Susskind, ...)*
- *Fuzzi ball, Firewall (Mathur, Braunstein, AMPS, ...)*

(5) ***Zero-Point Fluctuation Flow***

(Wilczek, Hotta-Schützhold-Unruh, Hawking (2015))

Gravitational zero energy states with supertranslation charges

~MASSAGE (1)~

Canonical typicality for non-vanishing Hamiltonians yields non-maximal entanglement among black holes and the Hawking radiation,
which makes spacetimes smooth without breaking monogamy.
Thus, no reason to have BH firewalls.

~MASSAGE (2)~

Typical states must be Gibbs states for smaller quantum systems with very high precision. If we have stable Gibbs states for old Schwarzschild BH's (and small AdS BH's), the heat capacity must be positive. Actually, it is negative. Thus the states of BH evaporation are not typical!

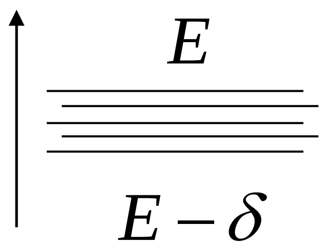
Inevitable Modification of the Page Curve

~MASSAGE (3)~

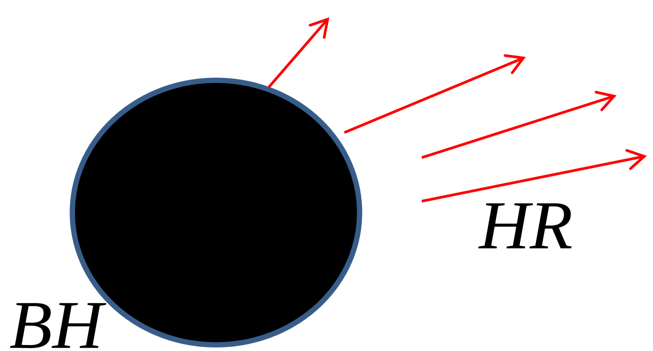
Microcanonical states $\rho \propto I_E$ ***are far from typical for finite-temperature old BH's, even though canonical states are typical and the large entropy O(V) is merely different from the microcanonical entropy by O(1). Entropy difference between a typical state and the canonical state must be exponentially small!***

$$\left| S_{typical} - S_{thermal} \right| \le O\left(\exp(-\gamma V)\right) \sim 0$$

You *cannot* use the microcanonical state in the typicality argument for evaporating black holes.



$$\cancel{\rho_{BH} \propto I_E}$$



Plan of this talk

I. Brief Review of Quantum Entanglement

II. Lubkin-Lloyd-Pagels-Page Theorem,
Page Curve Hypothesis
and BH Firewall Conjecture

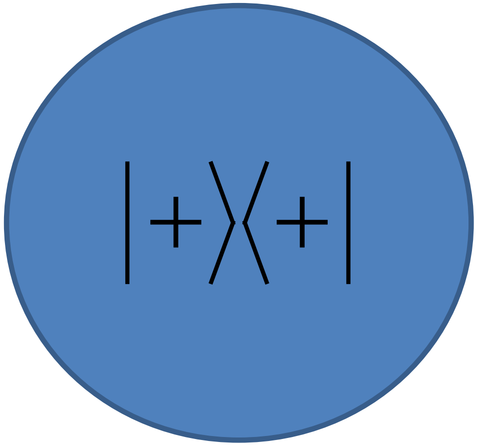
III. Canonical Typicality
for Non-Vanishing Hamiltonians
Yields No Firewalls

IV. Summary

I. Brief Review of Quantum Entanglement

Quantum entanglement is a correlation, which cannot be generated by local operations and classical communication for many-body systems and quantum fields.

As an example, let us consider a ½ spin system in the up state of the z component of the spin for simplicity.



Let us assume that any quantum operation Γ for the system is available.

Then, from this initial state, any quantum state of the system can be generated:

$$\rho = \Gamma\left[|+\rangle\langle+|\right] = \sum_{s=\pm} p_s |u_s\rangle\langle u_s|.$$

This can be proven in the following way.

For the up state $|+\rangle$, we first measure an observable given by

$$\boxed{\sigma = |\xi_+\rangle\langle\xi_+| - |\xi_-\rangle\langle\xi_-|.} \quad \left(\langle\xi_+|\xi_-\rangle = 0\right)$$

$$|\xi_+\rangle = \begin{bmatrix} \sqrt{p_+} \\ -\sqrt{p_-} \end{bmatrix}, \quad |\xi_-\rangle = \begin{bmatrix} \sqrt{p_-} \\ \sqrt{p_+} \end{bmatrix}$$

$$\left(p_\pm \geq 0, \quad p_+ + p_- = 1\right)$$

$$|+\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad |-\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Then, eigenvalue ± 1 of σ emerges with probability $p_\pm$.

Thus, the average state is given by

$$\Gamma_1\left[|+\rangle\langle+|\right] = p_+|\xi_+\rangle\langle\xi_+| + p_-|\xi_-\rangle\langle\xi_-|$$

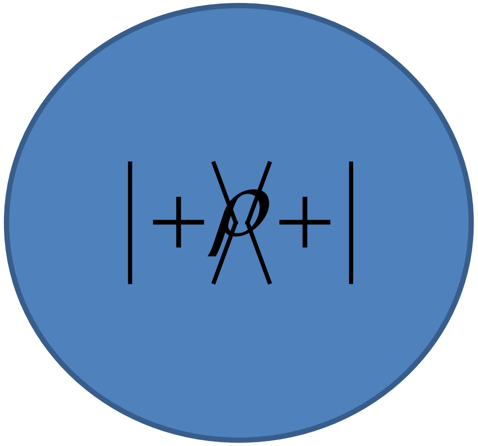
Next, let us perform a unitary operation which satisfies

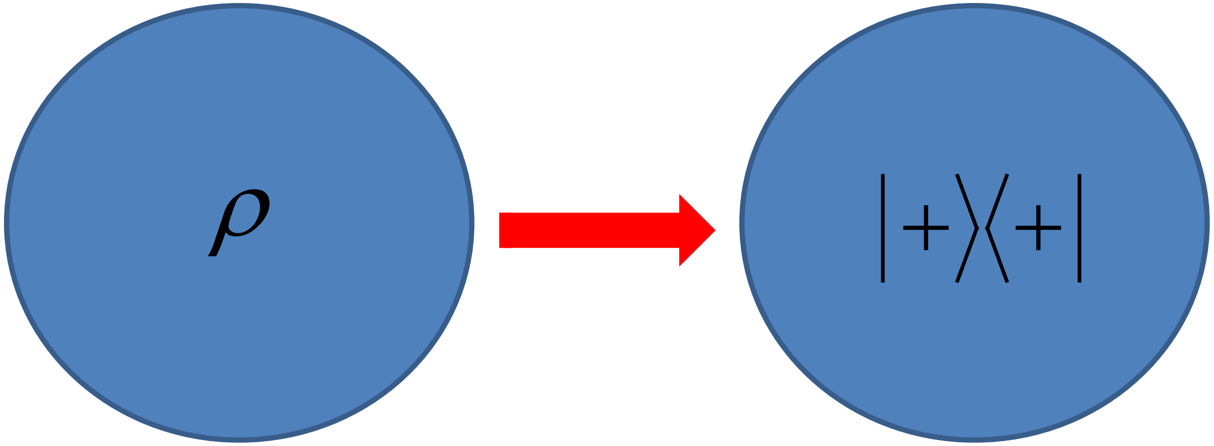
$$U\left|\xi_{\pm}\right\rangle = \left|u_{\pm}\right\rangle.$$

This yields the final state which we request.

$$\Gamma_2\left[\Gamma_1\left[\left|+\right\rangle\left\langle+\right|\right]\right] = p_+ U\left|\xi_+\right\rangle\left\langle\xi_+\right|U^{\dagger} + p_- U\left|\xi_-\right\rangle\left\langle\xi_-\right|U^{\dagger}$$

$$= p_+\left|u_+\right\rangle\left\langle u_+\right| + p_-\left|u_-\right\rangle\left\langle u_-\right| = \rho.$$





The inverse process can be also achieved. Let us measure the z component of the spin. Then, the state collapses to one of the eigenstates.

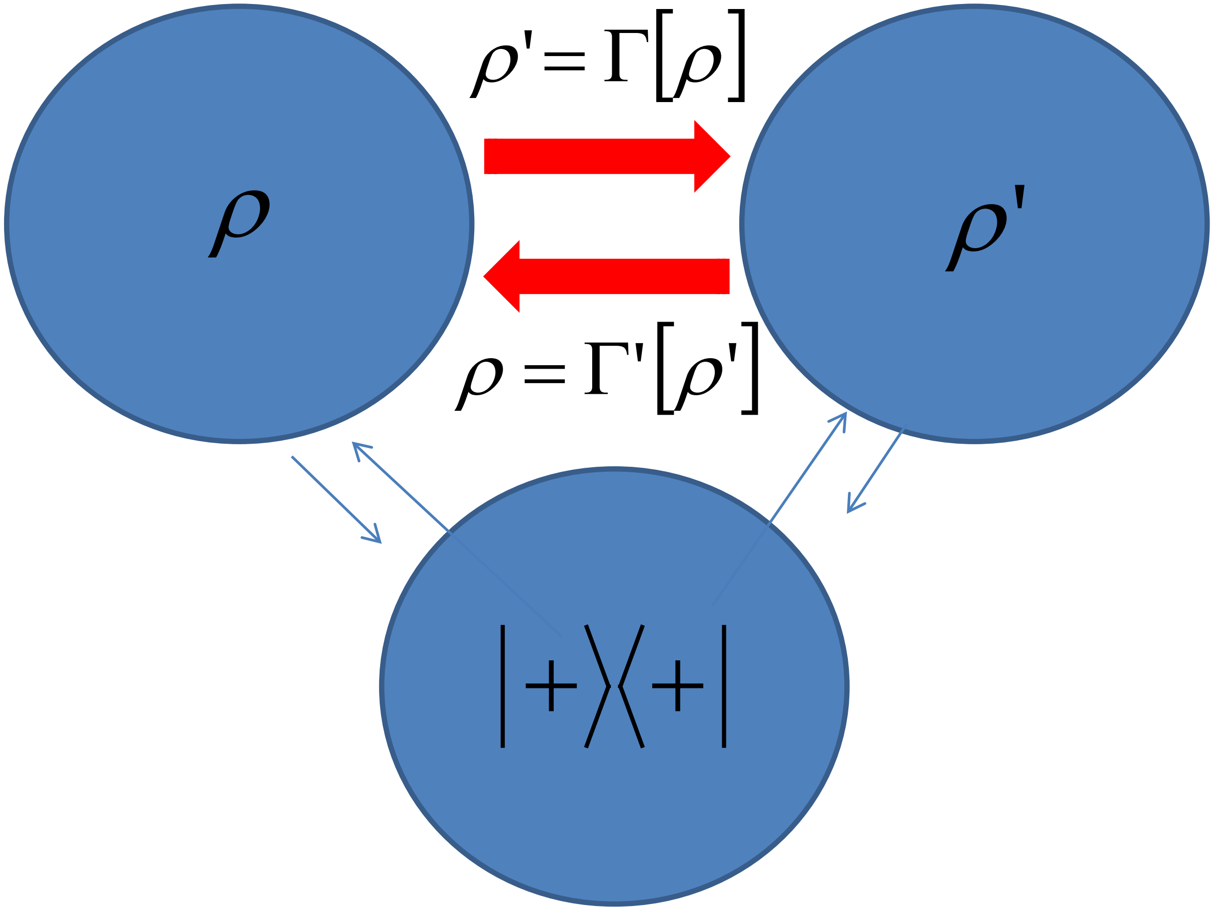
$$\rho \Rightarrow |\pm\rangle\langle\pm| \quad \textbf{with probability} \quad q_{\pm} = \langle\pm|\rho|\pm\rangle$$

If the down state emerges, the spin flip operation is performed so as to get the up state.

$$\sigma_x|-\rangle\langle-|\sigma_x^{\dagger} \Rightarrow |+\rangle\langle+|$$

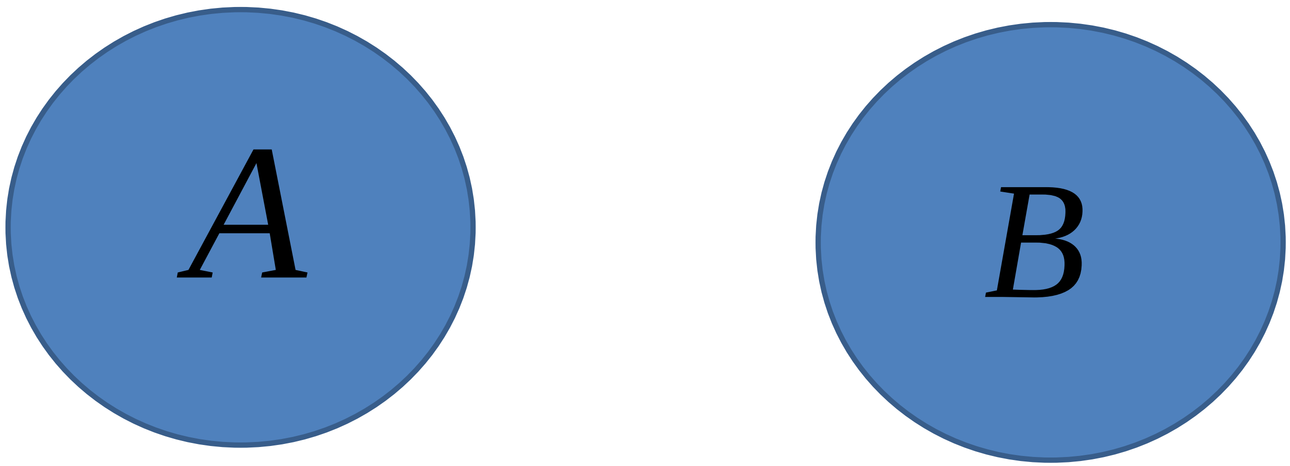
Therefore the state ρ is transformed into the up state with unit probability.

Therefore, any quantum state can be transformed into an arbitrary different state by quantum operations.



This result might sound trivial, but by using a similar argument for many-body systems, very interesting facts are revealed.
The concept of entanglement is one of them, as seen next.

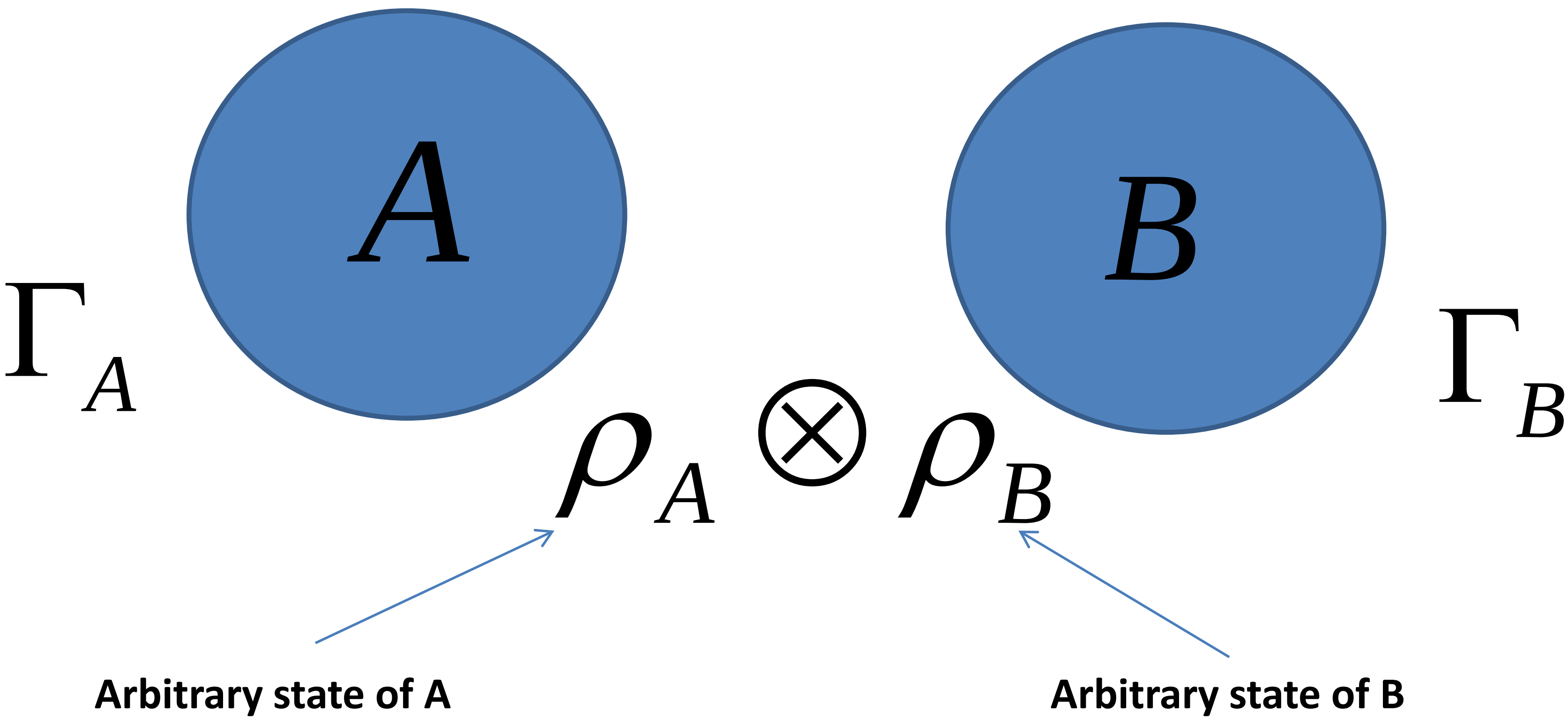
Entanglement is an index which describes a sort of quantum correlation between quantum systems. In order to grasp the concept of entanglement, let us next consider two ½ spin systems A and B.



$$\left|+_A\right\rangle\left\langle+_A\right|\otimes\left|+_B\right\rangle\left\langle+_B\right|$$

Let us assume that each state is pure and the up state at the first stage.

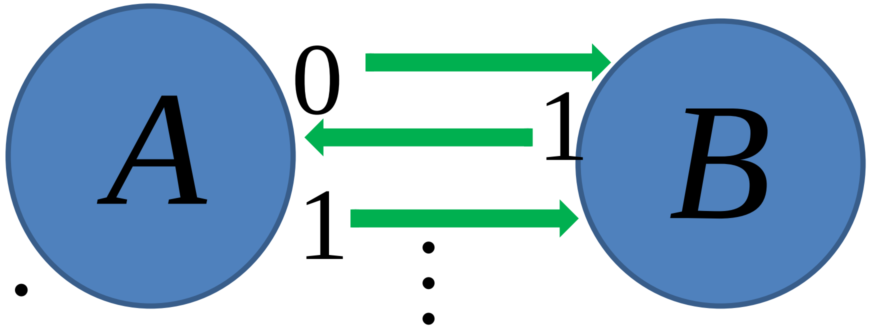
From the result proven previously, it turns out that local operation to each system transforms the initial pure state just into a product of mixed states without any correlation.



If classical communication is also allowed, we can show that the composite system gets classical correlation.
(The term “classical” is used in the context of communication theory.)

Local Operations

$$\Gamma_{A1}, \Gamma_{A2}, \cdots$$



Local Operations

$$\Gamma_{B1}, \Gamma_{B2}, \cdots$$

Classical Communication

Local Operations and Classical Communication (LOCC)

Available State Change:

$$\left|+_A\right\rangle\left\langle+_A\right| \otimes \left|+_B\right\rangle\left\langle+_B\right|$$

$$\Rightarrow \rho_{AB} = \sum_{\mu=1}^{\infty} p(\mu)\rho_A(\mu) \otimes \rho_B(\mu)$$

(called Seperable State)

$$p(\mu) \geq 0, \sum_{\mu=1}^{\infty} p(\mu) = 1$$

Arbitrary state of A

Arbitrary state of B

Simple Example of Generating a Separable State with Correlation:

σ_A

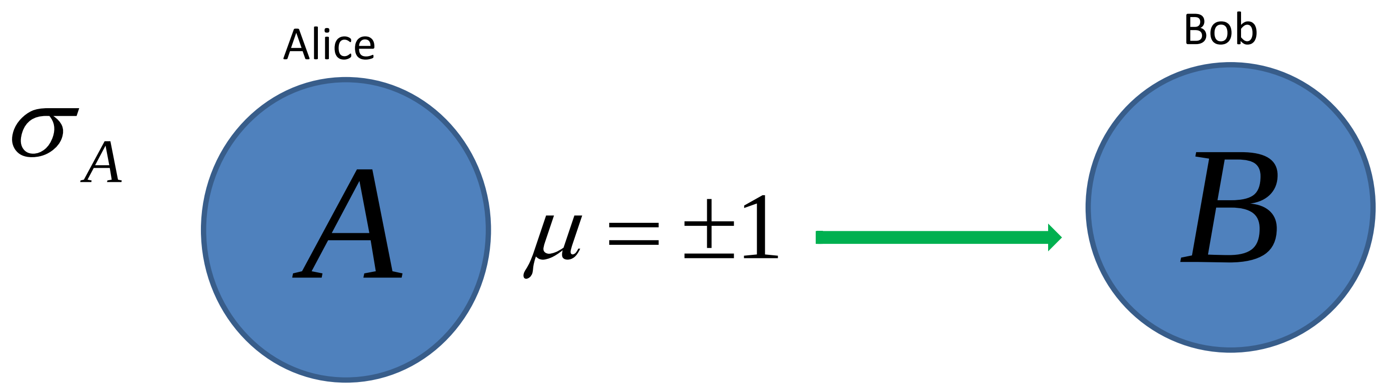
$$|+_A\rangle\langle +_A| \otimes |+_B\rangle\langle +_B|$$

At first, Alice measures σ_A for the system A in the initial state and obtains the eigenvalue $p_\pm$ with probability ± 1.

$$\sigma_A = \sum_{\mu=\pm 1} \mu |\xi_{\mu A}\rangle\langle \xi_{\mu A}|$$

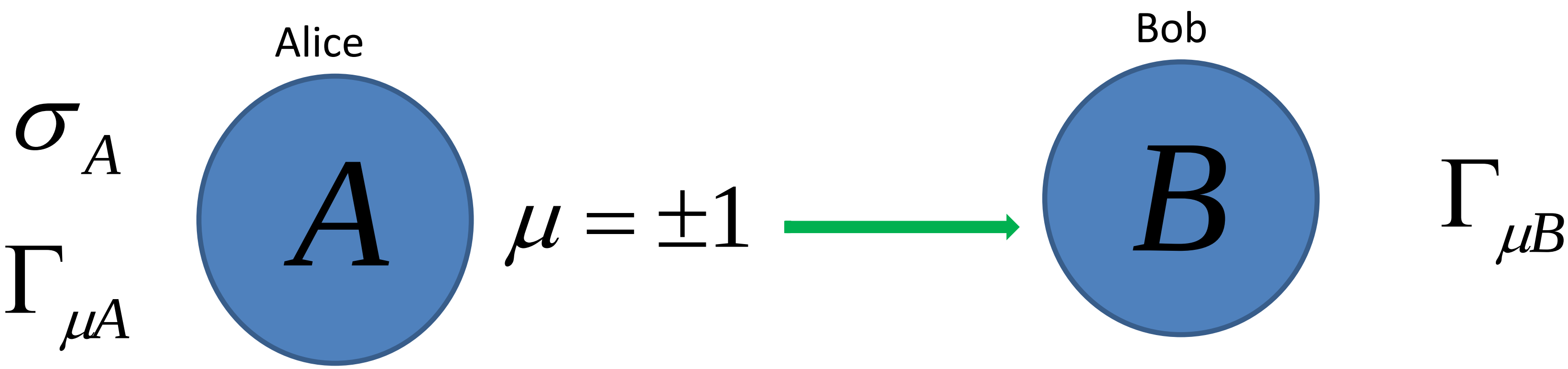
$$|\xi_{+A}\rangle = \begin{bmatrix} \sqrt{p_+} \\ -\sqrt{p_-} \end{bmatrix}, \quad |\xi_{-A}\rangle = \begin{bmatrix} \sqrt{p_-} \\ \sqrt{p_+} \end{bmatrix}$$

$$\left(p_\pm \geq 0, \quad p_+ + p_- = 1\right)$$



$$\left|\xi_{\mu A}\right\rangle\left\langle\xi_{\mu A}\right|\otimes\left|+_B\right\rangle\left\langle+_B\right|$$

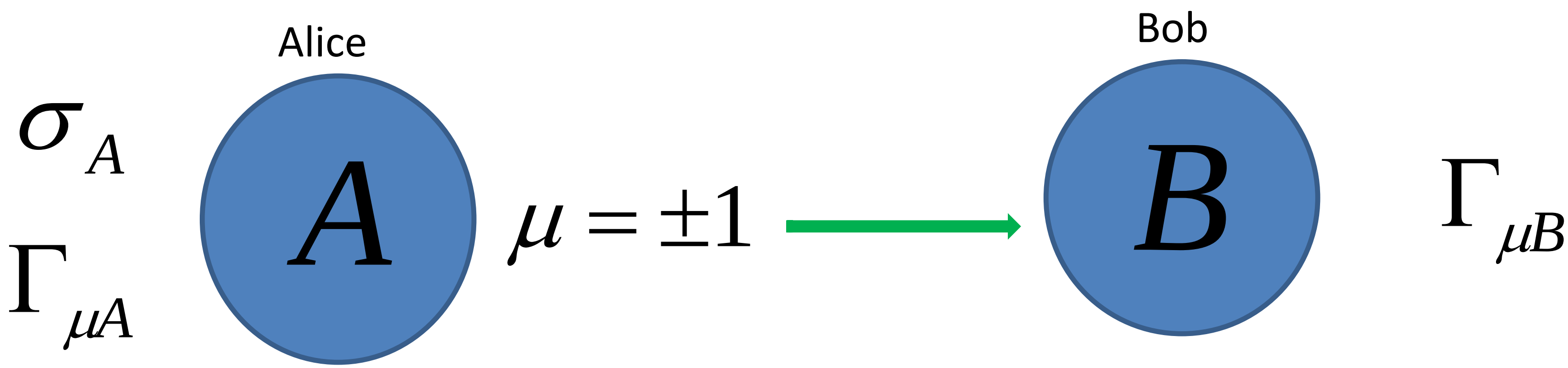
Next Alice announces the obtained eigenvalue $\mu = \pm 1$ **to Bob via a classical channel.**



$$\Gamma_{\mu A}\left[|\xi_{\mu A}\rangle\langle\xi_{\mu A}|\right]\otimes\Gamma_{\mu B}\left[|+_B\rangle\langle+_B|\right]$$

Finally, Alice and Bob perform arbitrary local operations **dependent on the eigenvalue** μ **for their systems.**

$$\rho_A(\mu)=\Gamma_{\mu A}\left[|\xi_{\mu A}\rangle\langle\xi_{\mu A}|\right],\ \rho_B(\mu)=\Gamma_{\mu B}\left[|+_B\rangle\langle+_B|\right]$$



$$\sum_{\mu=\pm} p_\mu \rho_A(\mu) \otimes \rho_B(\mu)$$

Then, the average state of the composite system becomes
a separable state with correlation.

Because LOCC is just a classical process in communication, entanglement between A and B is defined as a quantum effect such that the amount of entanglement never increases by LOCC. Besides, a product state of pure states has minimum entanglement and the value is zero.

This definition leads to a fact that
any separable state has zero entanglement.

$$ENT_{AB}\left(|+_A\rangle\langle +_A|\otimes|+_B\rangle\langle +_B|\right)=0$$

$$\Downarrow$$

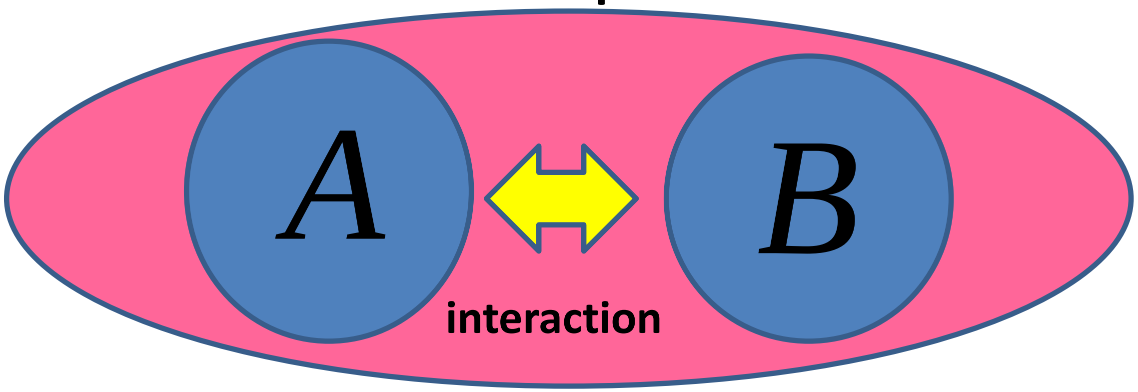
$$ENT_{AB}\left(\sum_{\mu=1}^{\infty}p(\mu)\rho_A(\mu)\otimes\rho_B(\mu)\right)=0.$$

Entangled states are defined as non-seperable states:

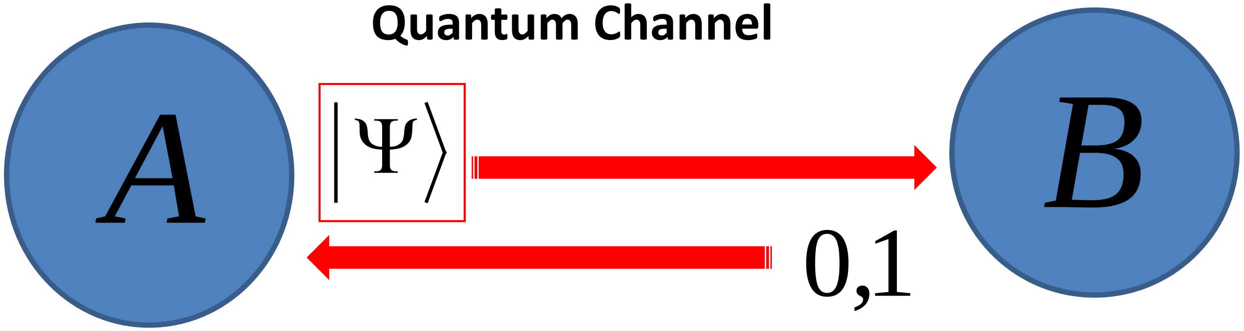
$$ENT_{AB}\left(\rho_{AB}\neq\sum_{\mu=1}^{\infty}p(\mu)\rho_A(\mu)\otimes\rho_B(\mu)\right)>0$$

Increasing entanglement requires non-local operations directly connecting A to B, or quantum channels which can transport not only classical but also quantum information (quantum states).

Non-Local Operation



Quantum Channel



Entanglement is a purely quantum effect and generates interesting phenomena, including break of Bell's inequality and quantum teleportation.

Example: entanglement of pure state of two systems

Contracted State:

$$\rho_A = \mathrm{Tr}_B\left[|\Psi_{AB}\rangle\langle\Psi_{AB}|\right],\ \rho_B = \mathrm{Tr}_A\left[|\Psi_{AB}\rangle\langle\Psi_{AB}|\right]$$

Entanglement Entropy

as one of entanglement indices

$$EE_{AB}\left(|\Psi_{AB}\rangle\langle\Psi_{AB}|\right) = -\mathrm{Tr}_A\left[\rho_A \ln \rho_A\right] = -\mathrm{Tr}_B\left[\rho_B \ln \rho_B\right]$$

Schmidt Decomposition of Pure State:

$$\left|\Psi_{AB}\right\rangle = \sqrt{p_{\Psi+}}\left|u_{A+}\right\rangle\left|v_{B+}\right\rangle + \sqrt{p_{\Psi-}}\left|u_{A-}\right\rangle\left|v_{B-}\right\rangle$$

$$p_{\Psi\pm} \geq 0, \sum_{s=\pm} p_{\Psi s} = 1, \left\langle u_{As}\middle|u_{As'}\right\rangle = \delta_{ss'}, \left\langle v_{Bs}\middle|v_{Vs'}\right\rangle = \delta_{ss'}$$

$$\Rightarrow EE_{AB}\left(\left|\Psi_{AB}\right\rangle\left\langle\Psi_{AB}\right|\right) = -\sum_{s=\pm} p_{\Psi s} \ln p_{\Psi s}$$

The Bell states attain the maximum value of entanglement entropy.

$$\left|Bell\right\rangle = \frac{1}{\sqrt{2}}\left[\left|+\right\rangle\left|+\right\rangle + \left|-\right\rangle\left|-\right\rangle\right]$$

$$\Rightarrow EE_{AB}\left(\left|Bell\right\rangle\left\langle Bell\right|\right) = \ln 2$$

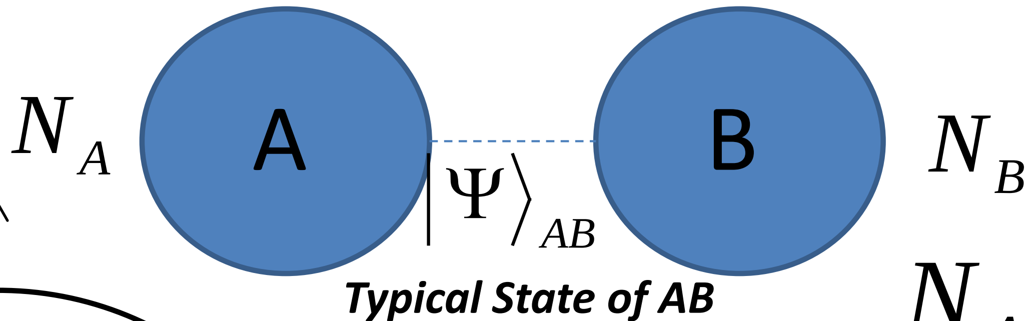
II. Lubkin-Lloyd-Pagels-Page Theorem, Page Curve Hypothesis and BH Firewall Conjecture

The BH firewall conjecture is based on the Page curve hypothesis,

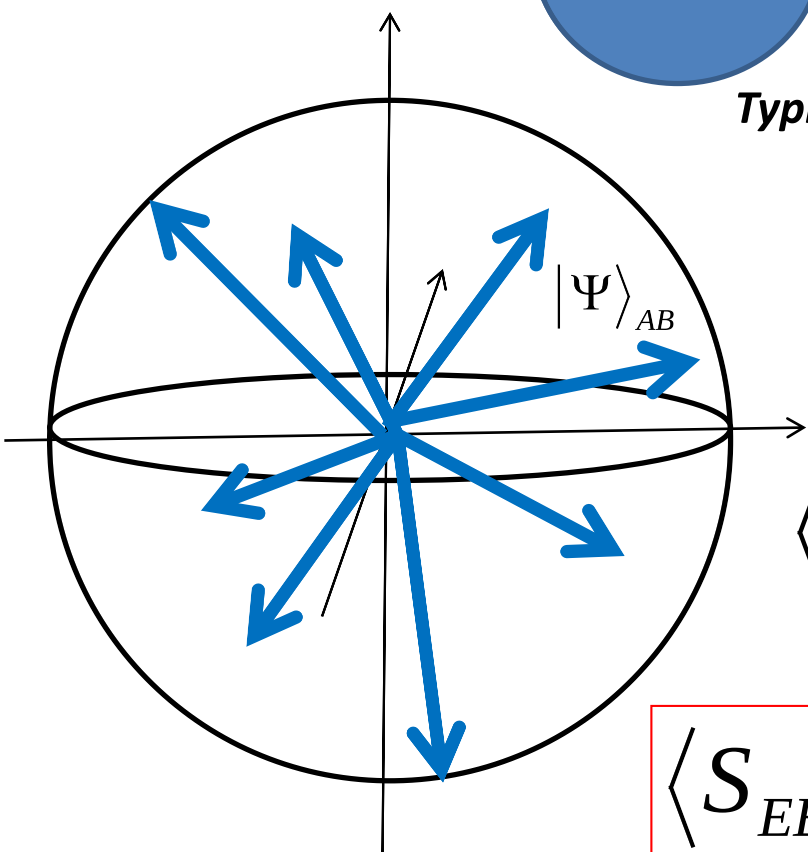
and the hypothesis was inspired by the Lubkin-Lloyd-Pagels-Page (LLPP) theorem.

Lubkin-Lloyd-Pagels-Page Theorem:

Typical states of A and B are almost maximally entangled when the systems are large.



$$N_A << N_B$$



$$\rho_A = Tr_B\left[|\Psi\rangle_{AB}\langle\Psi_{AB}|\right]$$

$$\langle S_{EE}\rangle = -\langle Tr_A[\rho_A \ln \rho_A]\rangle$$

$$\boxed{\langle S_{EE}\rangle \approx \ln N_A} \Rightarrow \boxed{\rho_A \approx \frac{1}{N_A} I_A}$$

Maximal Entanglement between A and B

$$\rho_A = \frac{1}{N_A} I_A \Rightarrow \boxed{\left|Max\right\rangle_{AB} = \frac{1}{\sqrt{N_A}} \sum_{n=1}^{N_A} \left|u_n\right\rangle_A \left|\tilde{v}_n\right\rangle_B}$$

Orthogonal unit vectors

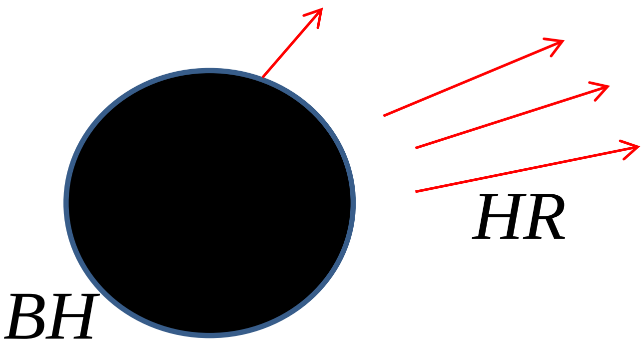
$$N_B \geq N_A$$

Let us assume that Hilbert-space dimensions of black holes and Hawking radiation become finite due to quantum gravity effect.

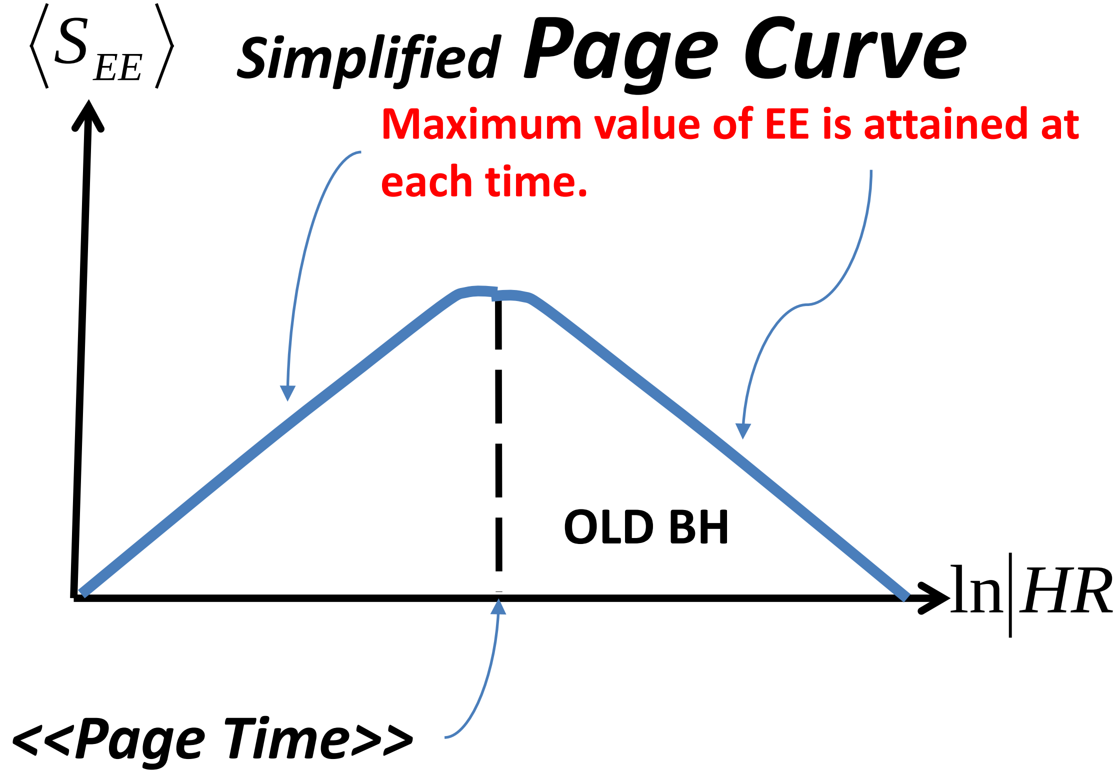
Page's Strategy for Finding States of BH Evaporation:

Nobody knows exact quantum gravity dynamics.

So let's gamble that the state scrambled by quantum gravity is one of TYPICAL pure states of the finite-dimensional composite system! That may not be so bad!



Simplified Page Curve



$$|BH| = \dim H_{BH},$$

$$|HR| = \dim H_{HR}$$

- $\ln|BH| \approx \ln|HR|$
- $M_{page} \approx 0.7 M_{bh}$

$$1 << |BH| << |HR| \Rightarrow \langle S_{EE} \rangle \approx \ln|BH| = \frac{A_{BH}}{4G}$$

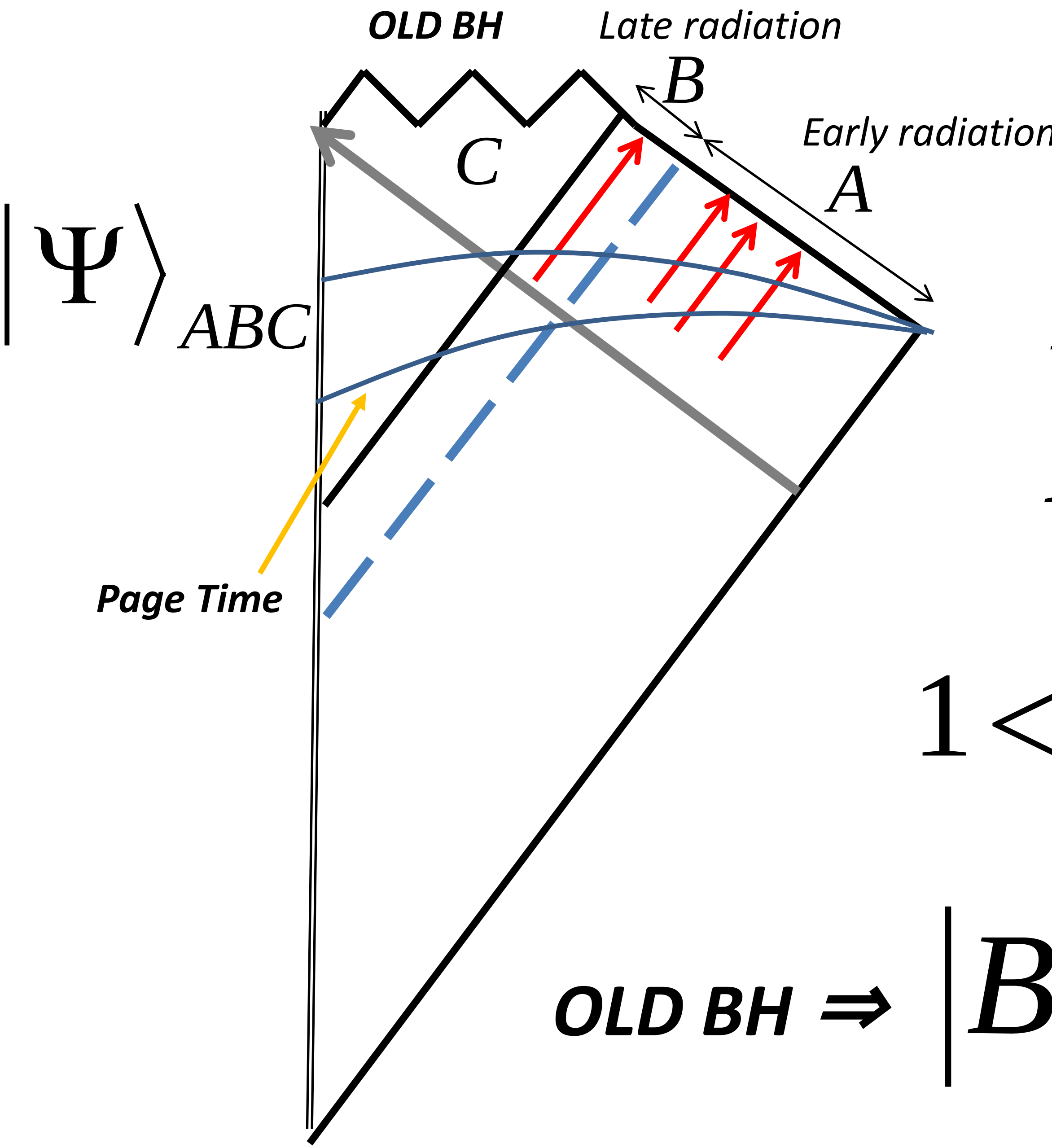
Page Curve Hypothesis for BH Evaporation:

Proposition I:

When the dimension of the BH Hilbert space is much larger or less than that of Hawking radiation, BH and HR in a typical pure state of quantum gravity share almost maximal entanglement. In other words, quantum states of the smaller system is almost proportional to the unit matrix.

Proposition II:

$$S_{EE} = S_{thermal}$$ *of the smaller system.*



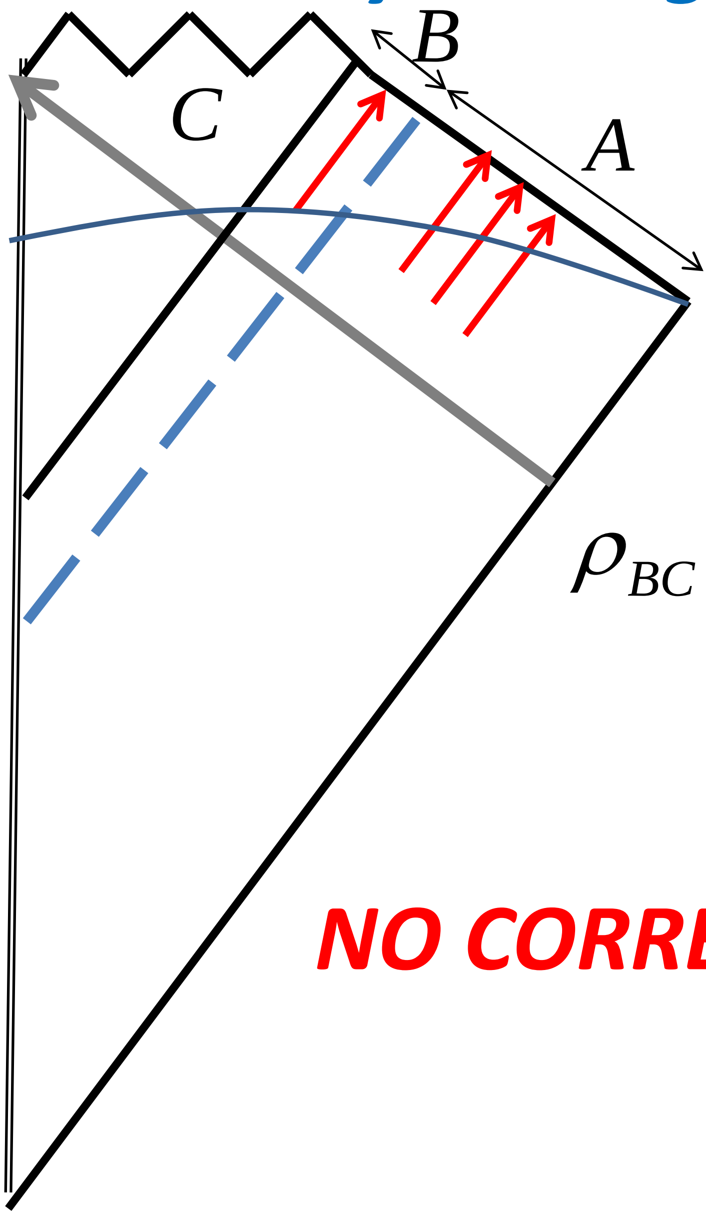
$$HR = A \cup B$$

$$BH = C$$

$$1 << |A|, |B|, |C|$$

OLD BH ⇒ $$|B||C| << |A|$$

Proposition I means that A and BC are almost *maximally entangled* with each other.



$$\rho_{BC} = \frac{1}{|BC|} I_{BC} = \left(\frac{1}{|B|} I_B \right) \otimes \left(\frac{1}{|C|} I_C \right)$$

NO CORRELATION BETWEEN B AND C!

Harrow-Hayden

$$\rho_{BC} = \left(\frac{1}{|B|} I_B\right) \otimes \left(\frac{1}{|C|} I_C\right)$$

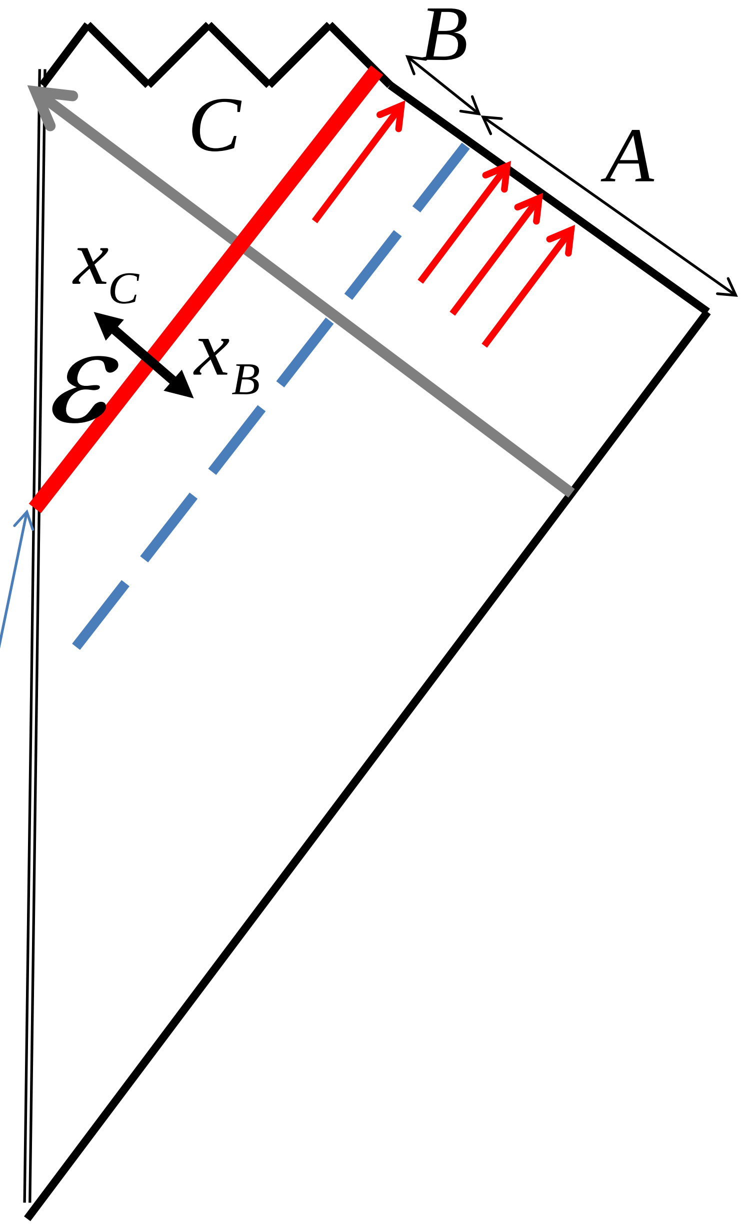
$$\Downarrow$$

$$Tr\left[\left(\frac{\varphi(x_B) - \varphi(x_C)}{\varepsilon}\right)^2 \rho_{BC}\right] = O\left(\frac{1}{\varepsilon^2}\right)$$

$$\varepsilon \to 0$$

$$\boxed{Tr\left[\left(\partial\varphi(x)\right)^2 \rho_{BC}\right] = \infty}$$

FIREWALL!



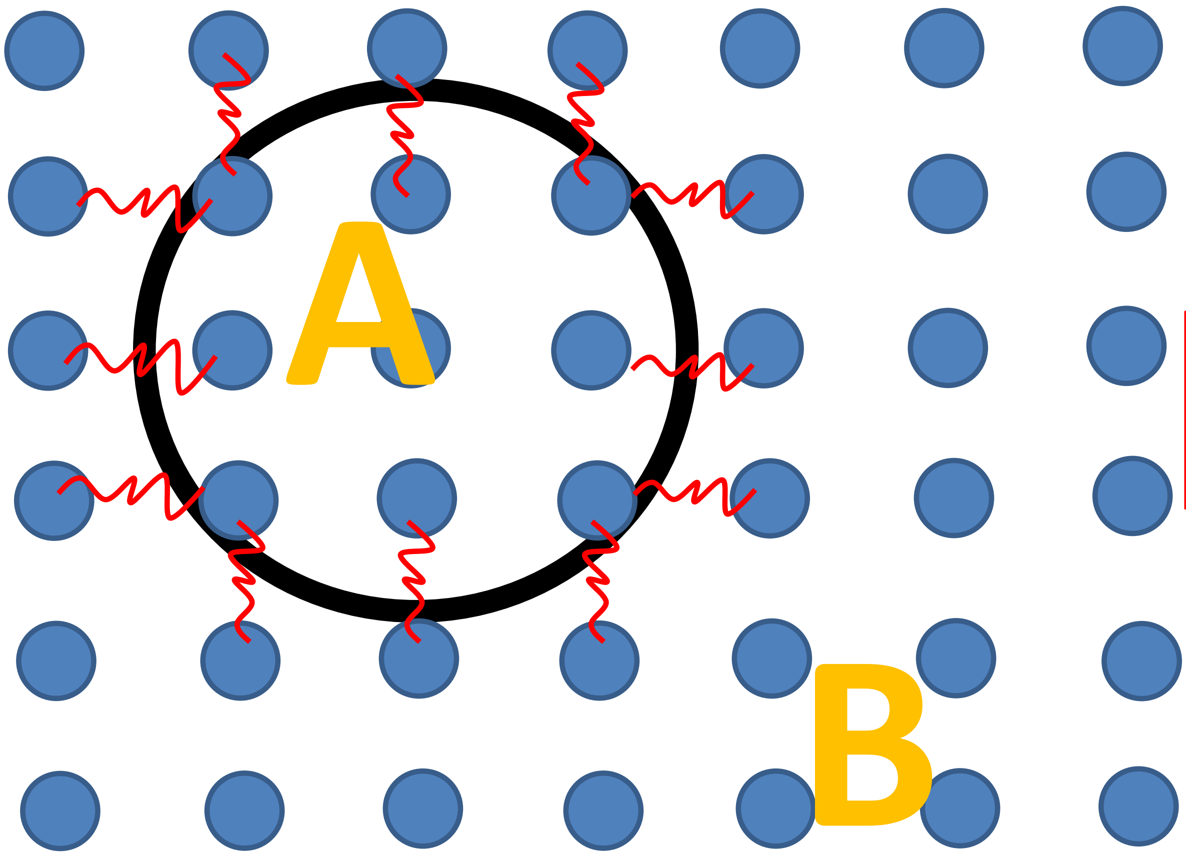
III. Canonical Typicality for Non-Vanishing Hamiltonian, and No Firewalls

Problem for Proposition I of Page Curve Hypothesis:

The area law of entanglement entropy is broken in a sense of ordinary many body physics, though outside-horizon energy density in BH evaporation is much less than the Planck scale.

$$S_{EE} \propto |\partial A| = |\partial B|$$

← *standard area law of entanglement entropy*



$$|\Psi\rangle_{AB} \approx |0\rangle_{AB}$$

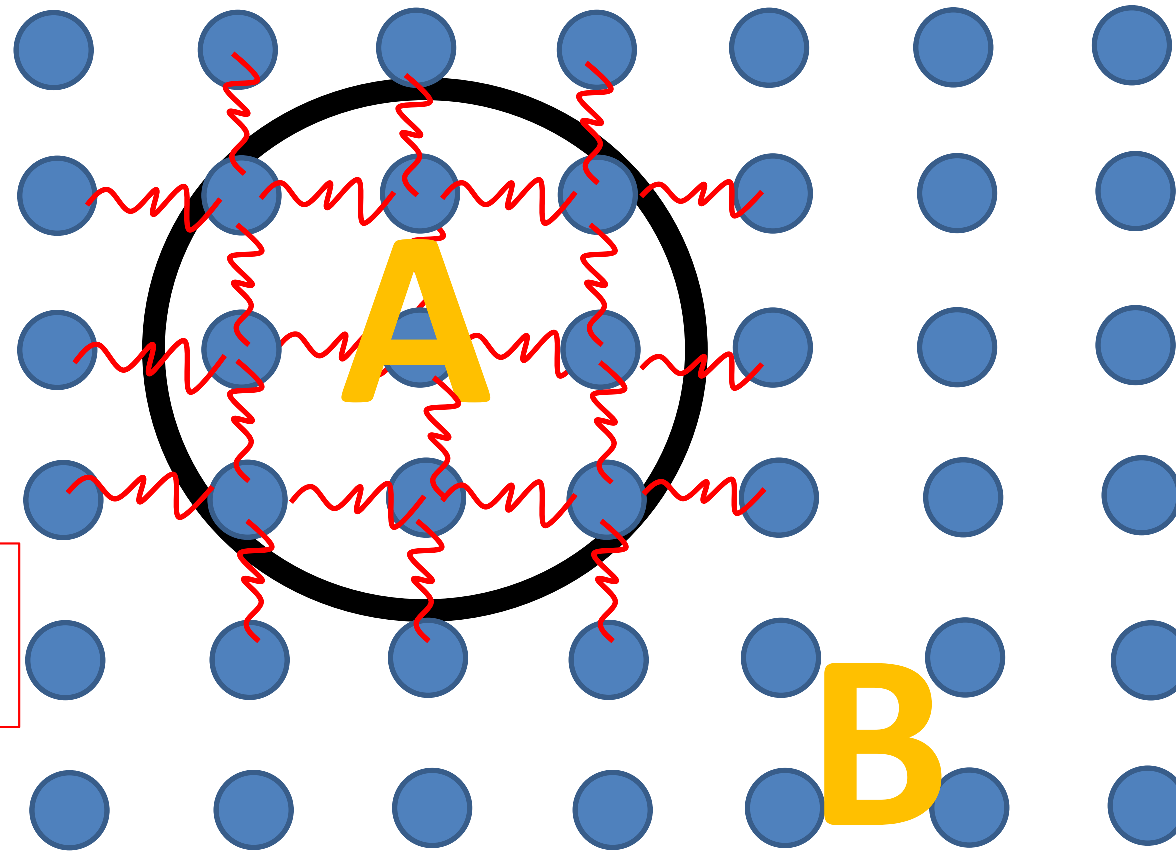
for low excited states

LLPP Typicality $\Rightarrow$

$$\left|\Psi\right\rangle_{AB} = \frac{1}{\sqrt{|A|}} \sum_{n=1}^{|A|} \left|u_n\right\rangle_A \left|\tilde{v}_n\right\rangle_B$$

Qubit network model

$$\left|A\right| = 2^{V_A}$$



$$\boxed{S_{EE} = \ln\left|A\right| \propto V_A}$$

Not area law,

but ***volume law for highly excited states!***

This is because zero Hamiltonian (complete degeneracy) is assumed in the LLPP theorem.
This is also an implicit premise of the Page curve hypothesis.

$$H_{AB} = 0.$$

In BH physics,
we have to treat canonical typicality with non-vanishing H in a precise manner.
Then non-maximal entanglement emerges and makes near-horizon regions smooth.
Thus no firewalls appear.

M. Hotta and A. Sugita, Prog. Theor. Exp. Phys, 123B04 (2015).

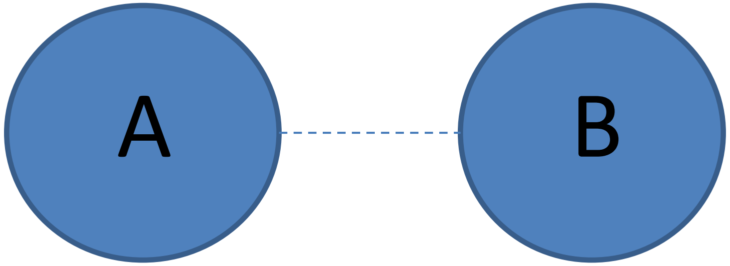
Microcanonical Energy Shell

(not a tensor product of the sub-Hilbert spaces)

$$H_{AB}\left|E_j\right\rangle = E_j\left|E_j\right\rangle$$

$$\Delta(E) = \left\{ j \mid E_j \in \left[E - \delta, E\right] \right\}$$

Microcanonical Energy Shell: $$V_{ES}(E) = \left\{ \sum_{j \in \Delta(E)} c_j \left|E_j\right\rangle \right\}$$

N_A 

N_B

$$|\Psi\rangle_{AB} \in V_{ES}(E)$$

$$\rho_A = Tr_B\left[|\Psi\rangle_{AB}\langle\Psi|_{AB}\right]$$

$$= \frac{1}{N_A}\left[I_A + \sum_{n=1}^{N_A{}^2-1}\langle T_n\rangle T_n\right]$$

Bloch Representation of higher-dim quantum states

$$T^{\dagger}{}_n = T_n,\ Tr\left[T_n\right] = 0,\ Tr\left[T_n T_{n'}\right] = N_A\delta_{nn'}$$

$$\boxed{\langle T_n\rangle = Tr\left[T_n\rho_A\right]}$$

Evaluate $\left\langle T_n \right\rangle$ ***for*** $\left|\Psi\right\rangle_{AB} = \sum_{j\in\Delta(E)} c_j \left|E_j\right\rangle$**.**

$$D = \dim V_{ES}(E) = \dim\left\{ \sum_{j\in\Delta(E)} c_j \left|E_j\right\rangle \right\}$$

$$N_A << N_B$$

Volume of B

Hilbert space dimension of B

$$D \propto \exp(\gamma V_B(N_B)) >> 1$$

for ordinary systems.

Uniform Ensemble on Mircrocanonical Energy Shell:

$$p(c) \propto \delta\left(\sum_{j \in \Delta(E)} \left| c_j \right|^2 - 1 \right) \qquad \int p(c) d^D p = 1$$

$$\overline{f} = \int f(c) p(c) d^D p$$

$$\overline{c_j c_{j'}^*} = \frac{1}{D} \delta_{jj'},$$

$$\overline{c_j c_k c_{j'}^* c_{k'}^*} = \frac{1}{D(D+1)} \left(\delta_{jj'} \delta_{kk'} + \delta_{jk'} \delta_{j'k} \right)$$

$$\overline{\left(\langle T_n\rangle - \overline{\langle T_n\rangle}\right)^2} \le \frac{\left\| T_n{}^2 \right\|}{D+1}$$

Max eigenvalue

$$\overline{Tr_A\left(\rho_A - \overline{\rho_A}\right)^2} \le \left(\frac{1}{N_A} \sum_{n=1}^{N_A{}^2-1} \left\| T_n{}^2 \right\| \right) \frac{1}{D+1}$$

N_B ***independent!***

$$D \propto \exp(\gamma V_B) = O\left(\exp\left(10^{23}\right)\right) >> 1$$

Sugita Theorem (2006)

$$\boxed{\left\| \rho_A - \overline{\rho_A} \right\| \le O(\exp(-\gamma V_B))}$$

Hotta-Sugita (2015) as a response to a BH firewall debate with Daniel Harlow

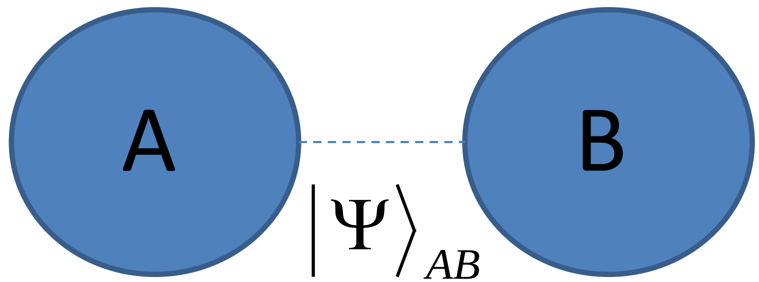
Private Communication with D. Harlow about "Jerusalem Lectures on Black Holes and Quantum Information", arXiv:1409.1231 .

Harlow argued a canonical typicality in a weak interaction limit.

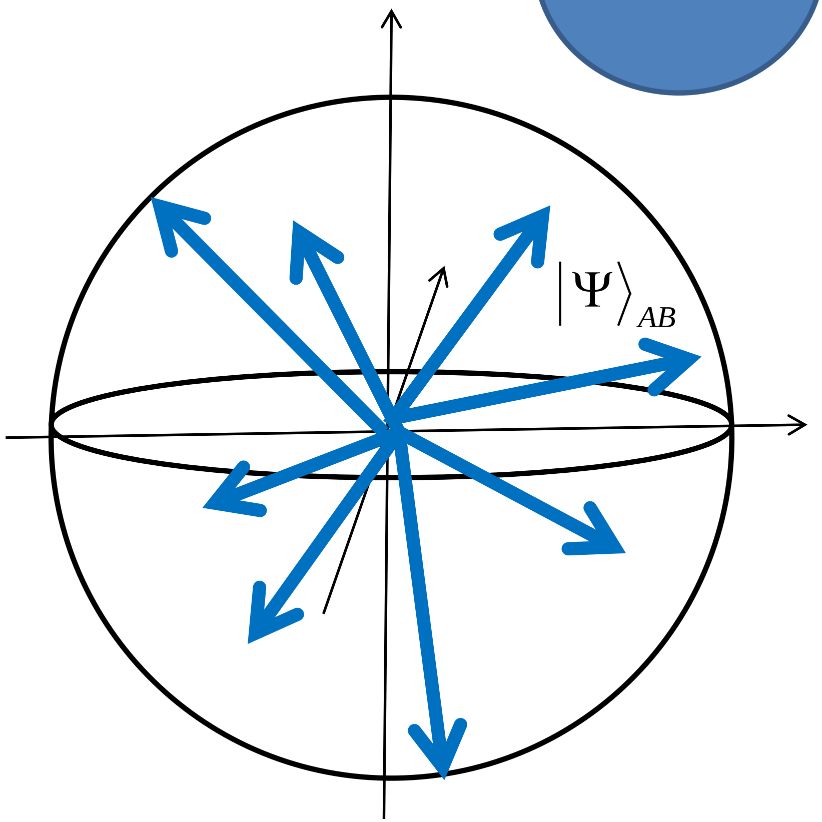
$$H = H_A + H_B + \underbrace{\cdots}_{\text{Negligibly small}}$$

$$\langle H \rangle = E_A + E_B = const.$$

$$E_A + E_B = const.$$



$$N_B >> N_A$$



$$\rho_A = Tr_B\left[|\Psi\rangle_{AB}\langle\Psi_{AB}|\right]$$

$$\langle S_{AB}\rangle = -\langle Tr_A[\rho_A \ln \rho_A]\rangle$$

$$\boxed{\rho_A \approx \frac{1}{Z_A}\exp(-\beta H_A)}$$

$$\boxed{\langle S_{AB}\rangle \approx S_{thermal,A}(\beta)}$$

Without any proof, Harlow argued these only in the weak interaction limit.

Harlow pointed out a possibility that BH firewalls may exist even after canonical typicality with non-zero Hamiltonian.

$$HR = A \cup B, \qquad BH = C$$

$$1 << |A|, |B|, |C| \quad |B||C| << |A|$$

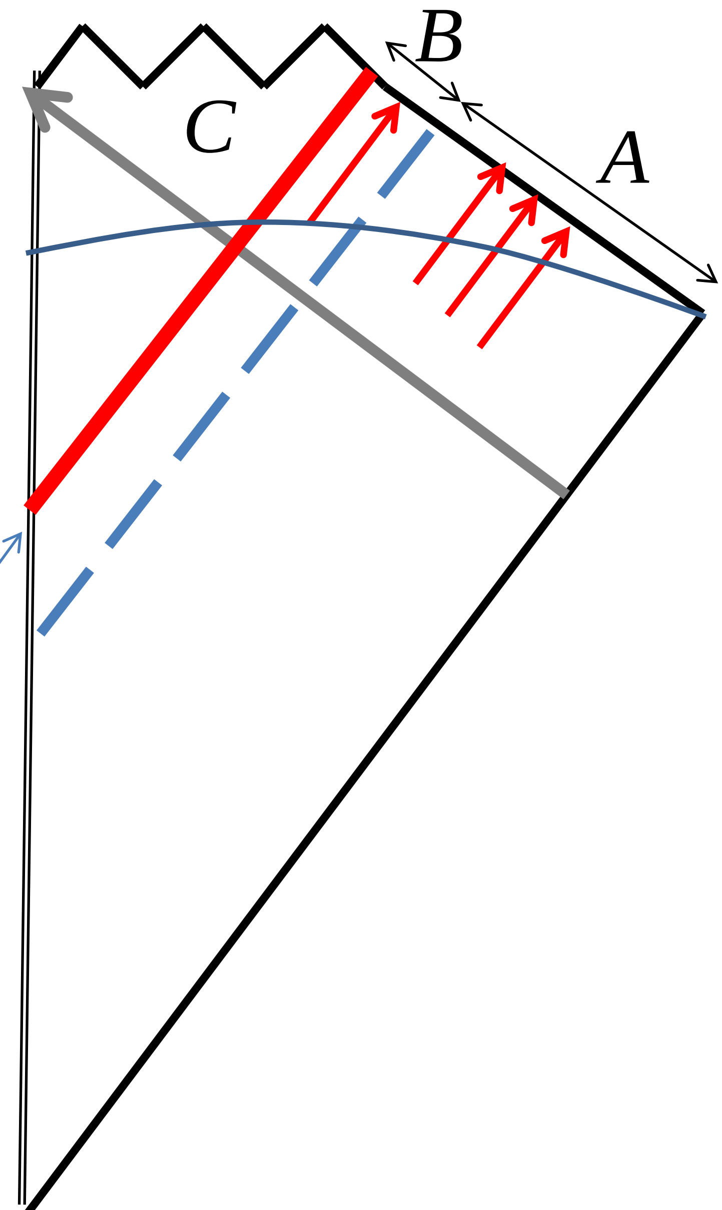
$$\rho_{BC} \propto \exp\left(-\beta\left(H_B + H_C\right)\right)$$
$$= \exp\left(-\beta H_B\right) \otimes \exp\left(-\beta H_C\right)$$

No Correlation, just like $I_B \otimes I_C$

$$\boxed{Tr\left[\left(\partial\varphi(x)\right)^2 \rho_{BC}\right] = \infty ?}$$

FIREWALL?

Harlow, [arXiv:1409.1231]



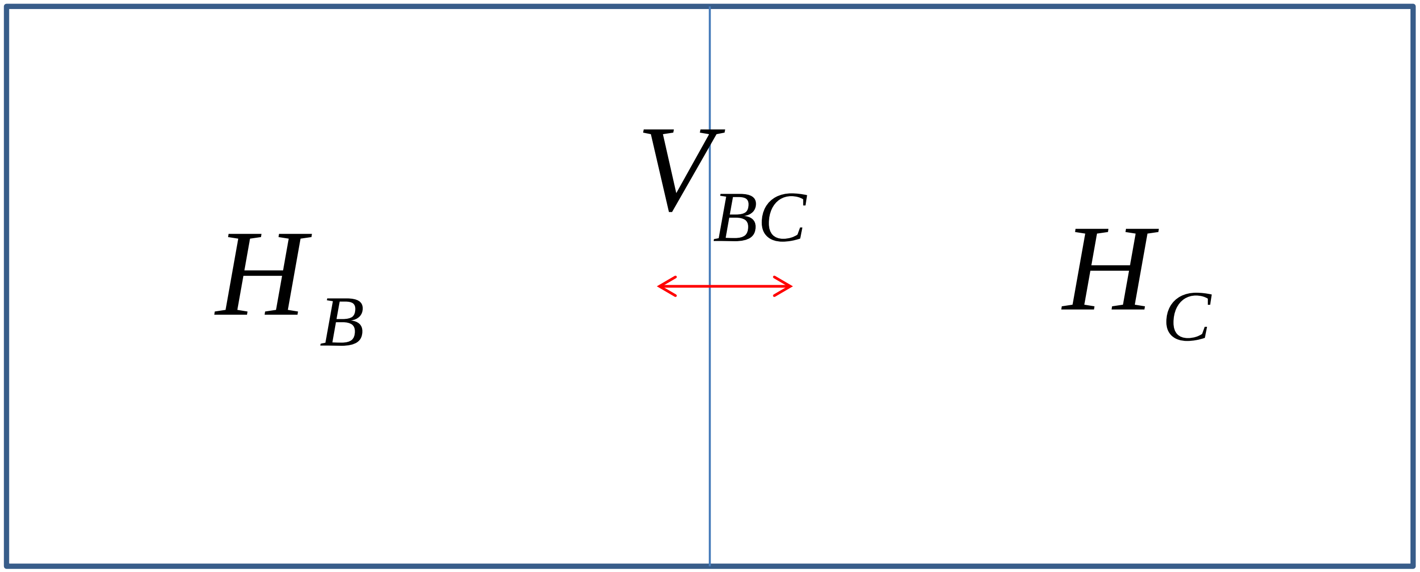
However, the worry is useless.
We can prove nonexistence of firewalls for general systems by using the general theory of canonical typicality. *M. Hotta and A. Sugita, Prog. Theor. Exp. Phys, 123B04 (2015).*

Irrespective of the strength of the interaction between B and C,

$$\rho_{BC} \propto \exp\left(-\beta\left(H_B + H_C + \underline{V_{BC}}\right)\right)$$

$$|B||C| << |A|$$

Actually, a correlation exists between B and C for small interactions.



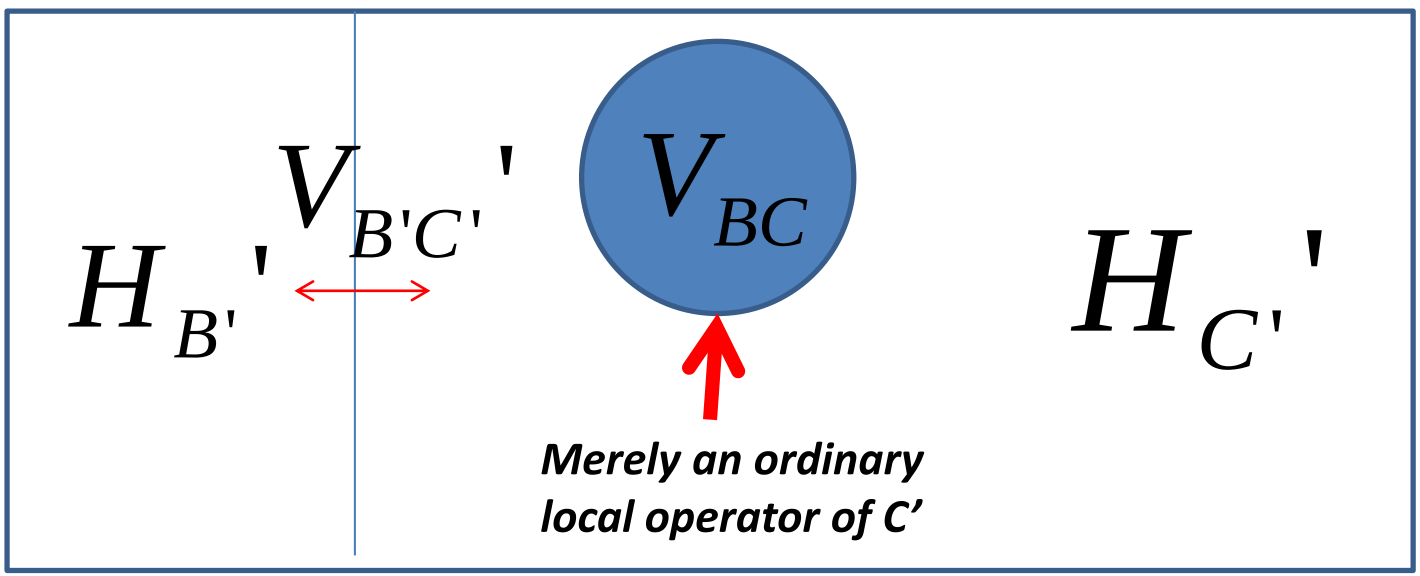
$$\rho_{BC} \propto \exp\left(-\beta\left(H_B + H_C + V_{BC}\right)\right)$$

Harlow's worry: $$\lim_{V_{BC}\to 0}\left|Tr\left[\rho_{BC}V_{BC}\right]\right| = \infty !?$$

Border shift does not change physics at all.

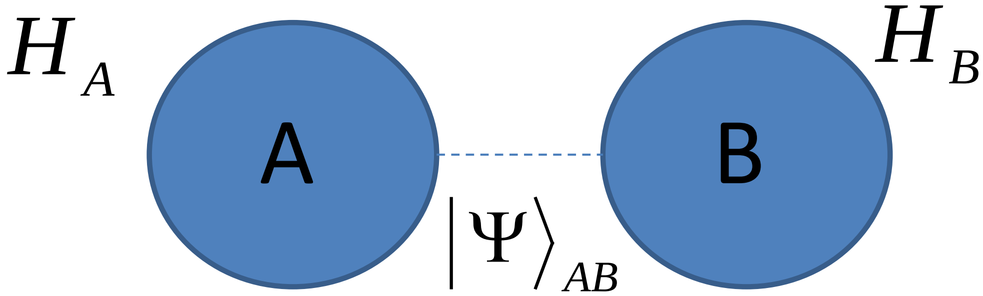
$$H_B + H_C + V_{BC} = H_{B'}' + H_{C'}' + V_{B'C'}'$$

$$\rho_{BC} = \rho_{B'C'}' \propto \exp\left(-\beta\left(H_{B'}' + H_{C'}' + V_{B'C'}'\right)\right)$$



$$\left|Tr\left[\rho_{BC} V_{BC}\right]\right| = \left|Tr\left[\rho_{B'C'}' V_{BC}\right]\right| < \infty$$ **No firewall!**

***Remark*: for ordinary weakly interacting quantum systems, entanglement entropy is upper bounded by thermal entropy, as long as stable Gibbs states exist.**



$$E_A + E_B = E$$

Arbitrary state: $\rho_A = Tr_B\left[|\Psi\rangle_{AB}\langle\Psi_{AB}|\right]$

Gibbs state: $\overline{\rho}_A = \exp(-\beta(E)H_A)/Z_A(\beta(E))$

$$\boxed{S_{EE} = -Tr\left[\rho_A \ln \rho_A\right] \leq -Tr\left[\overline{\rho}_A \ln \overline{\rho}_A\right] = S_{thermal}}$$

Conventional “proof”:

$$I = -Tr_A[\rho_A \ln \rho_A] - \lambda_1 (Tr_A[\rho_A H_A] - E_A) - \lambda_2 (Tr_A[\rho_A] - 1)$$

$$\delta I = 0$$

$$\bar{\rho}_A = \exp(-\beta H_A) / Z_A(\beta)$$

$$\boxed{-Tr[\rho_A \ln \rho_A] \le -Tr[\bar{\rho}_A \ln \bar{\rho}_A]}$$

***If a stable Gibbs state exists,* it attains the maximum of the von Neumann entropy with average energy fixed.**

Unfortunately, the typicality argument cannot be applied to Schwarzschild BH evaporation!

Actually, from our result,
the typical state must be a Gibbs state,
but…

No stable Gibbs state for Schwarzschild BH

due to negative heat capacity! ***(Hawking –Page, 1983)***

$$\langle E\rangle = M_{BH} = \frac{1}{8\pi GT} \Rightarrow \frac{d\langle E\rangle}{dT} = -\frac{1}{8\pi GT^2} < 0$$

If there exists a stable Gibbs state, heat capacity must be positive.

$$Z_{BH}(\beta) = Tr\left[\exp\left(-\beta H_{BH}\right)\right]$$

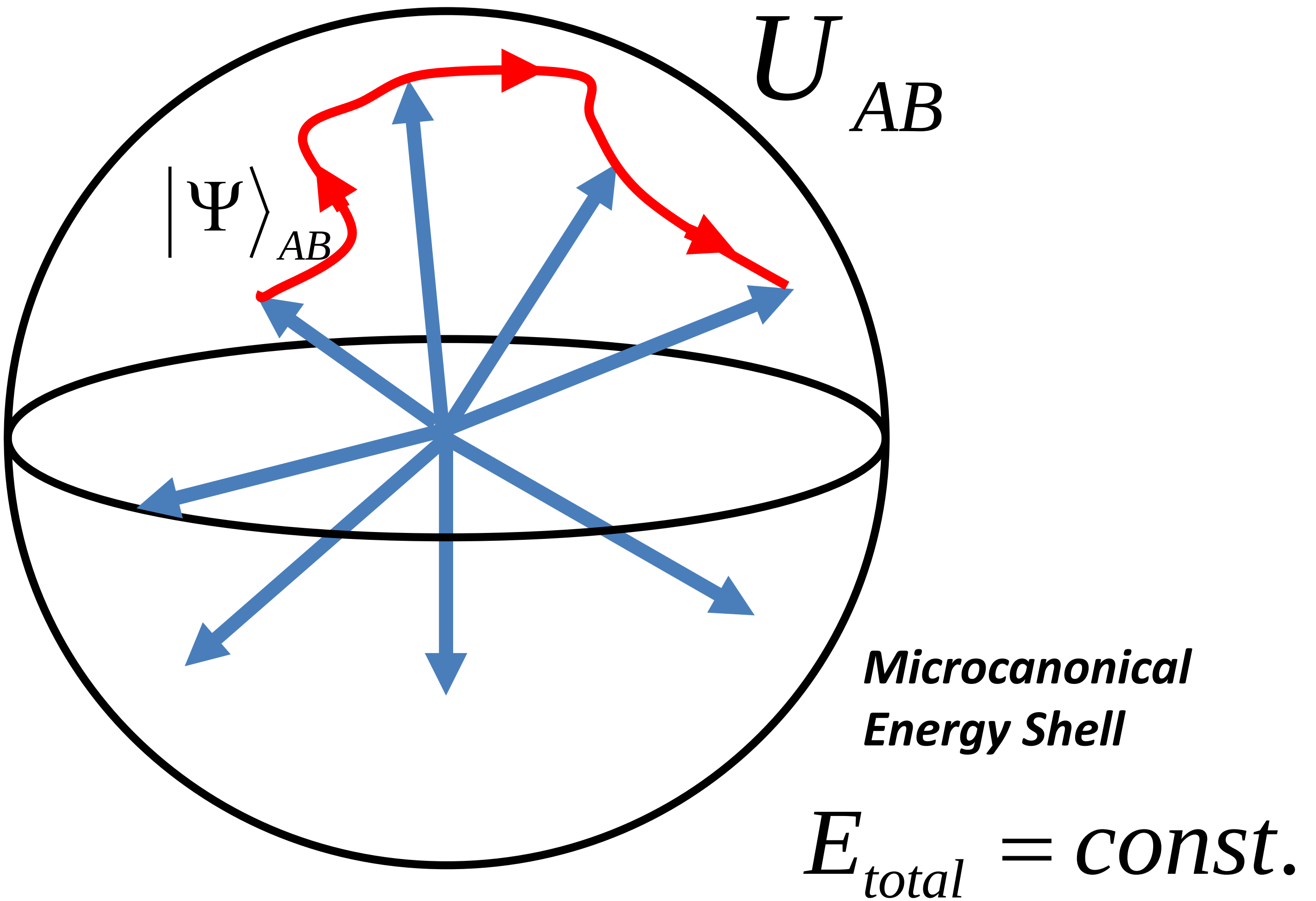
$$\Downarrow$$

$$\boxed{\frac{d\langle E\rangle}{dT} = \frac{\left\langle \left(E - \langle E\rangle\right)^2 \right\rangle}{T^2} > 0}$$

Thus, a system of a black hole and Hawking radiation is not in typical states, at least in the sense of the Page curve hypothesis, during BH evaporation. Because we have no stable Gibbs state," thermal entropy" of Schwarzschild BH ($A/(4G)$) is not needed to be a upper bound of entanglement entropy.

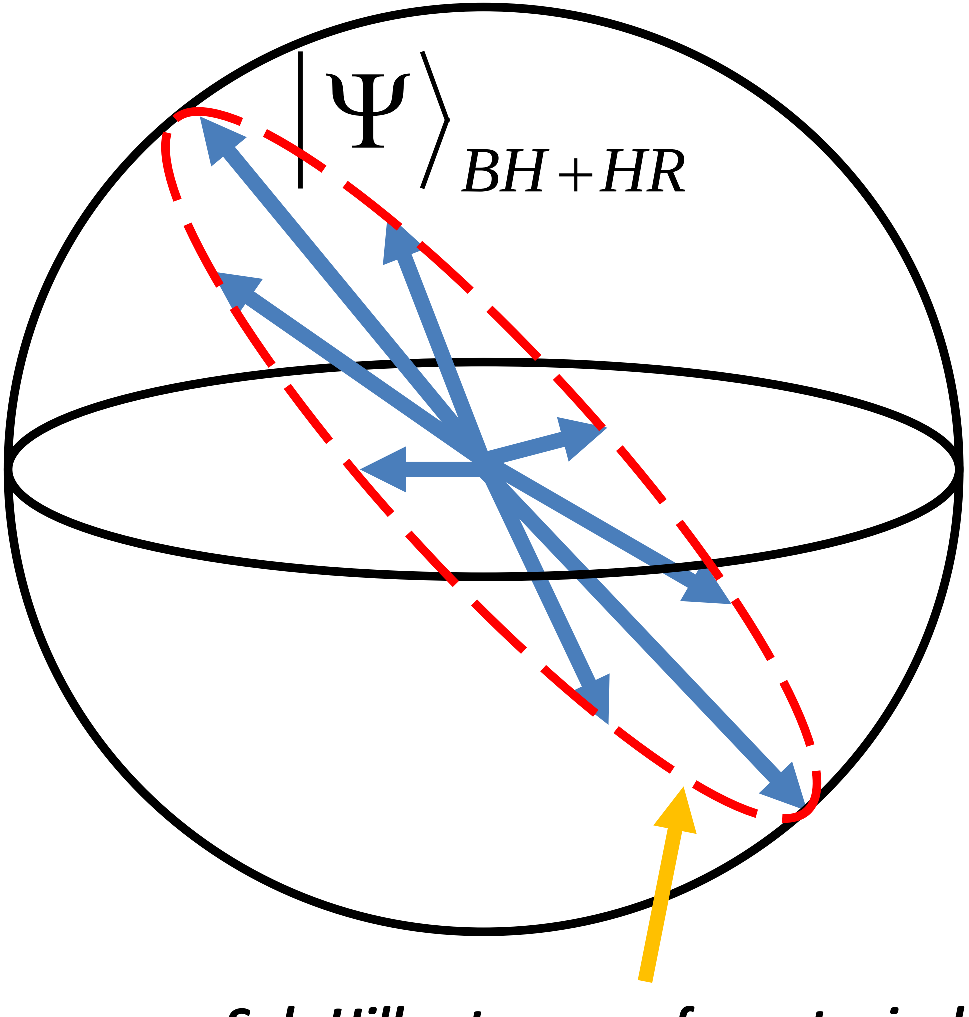
$$\cancel{S_{EE} \leq S_{thermal} \approx A/(4G)}$$

In ordinary quantum systems,



$|\Psi\rangle_{AB}$ ***is a typical state with almost certainty after a relaxation time.***

The state of BH evaporation can be non-typical until the last burst.



$$U_{BH} \otimes I_{HR}$$

Fast scrambling of BH does not contribute to entanglement between BH and HR.

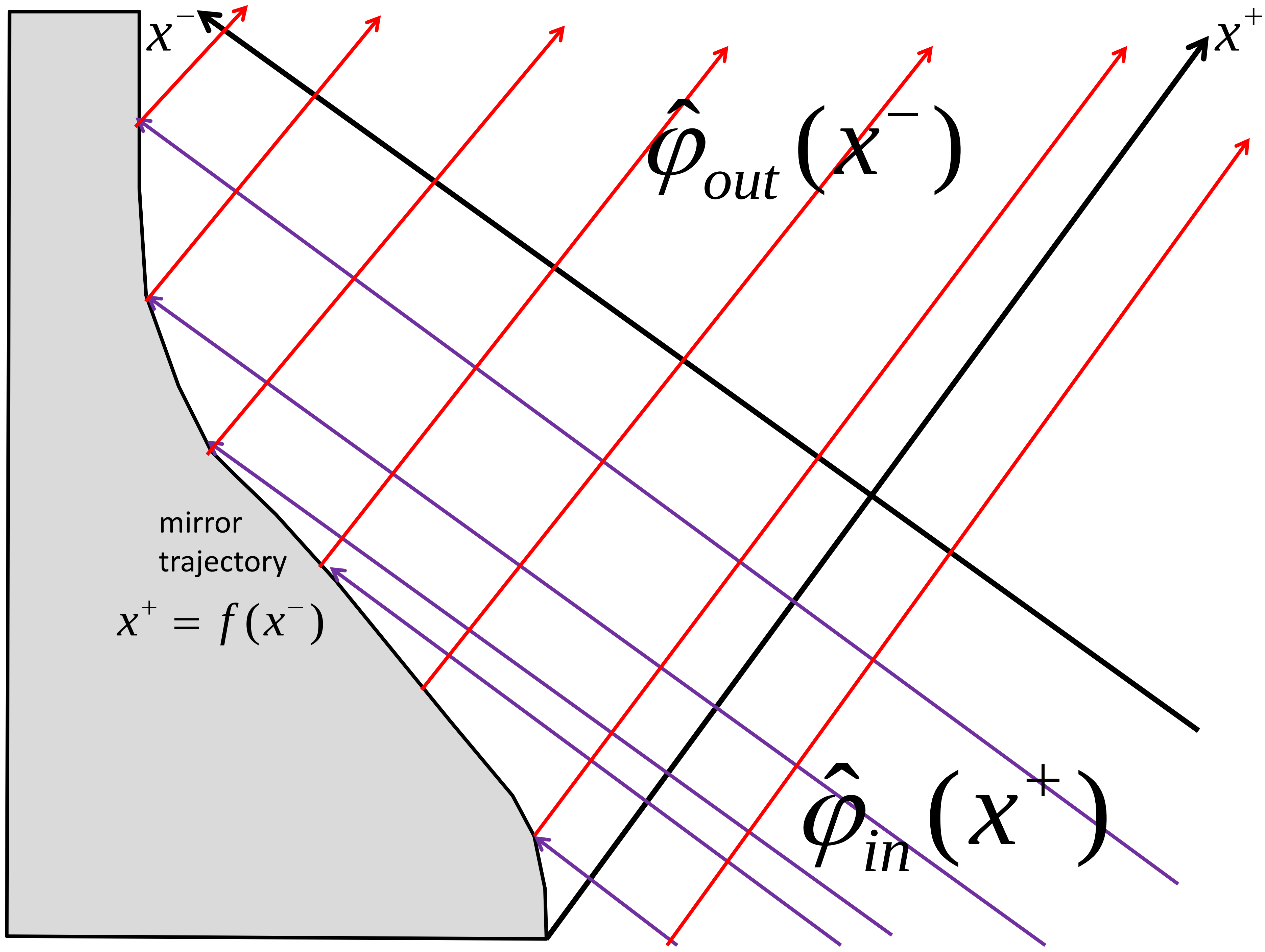
$$U^{(emission)}$$

Non-chaotic HR emission generated by smooth space time curvature outside horizon

If so, how is the Page curve modified?

***The moving mirror model* is totally unitary. So we are able to learn how the information can be retrieved.**

The model is a tool to explore the Page curve hypothesis and its modification by using various mirror trajectories.


x^-
x^+
$\hat{\varphi}_{out}(x^-)$
mirror trajectory
$x^+ = f(x^-)$
$\hat{\varphi}_{in}(x^+)$

Mirror Trajectory:

$$x^{+} = f(x^{-}) \qquad \left(x^{\pm} = t \pm x\right)$$

Boundary Condition:

$$\hat{\varphi}\,|_{x^{+}=f(x^{-})} = 0$$

Solution:

$$\hat{\varphi}\left(x,t\right) = \hat{\varphi}_{in}(x^{+}) - \hat{\varphi}_{in}(f(x^{-}))$$

Scattering Relation:

$$\hat{\varphi}_{out}(x^{-}) = \hat{\varphi}_{in}(f(x^{-}))$$

Out-going energy flux: $\hat{T}_{--} =: \partial_- \hat{\varphi} \partial_- \hat{\varphi} :=: \partial_- \hat{\varphi}_{out} \partial_- \hat{\varphi}_{out} :$

$$\boxed{\langle 0_{in} | \hat{T}_{--}(x^-) | 0_{in} \rangle = -\frac{1}{24\pi} \left[\frac{\partial_-{}^3 f(x^-)}{\partial_- f(x^-)} - \frac{3}{2} \left(\frac{\partial_-{}^2 f(x^-)}{\partial_- f(x^-)} \right)^2 \right]}$$

derived from $\hat{\varphi}_{out}(x^-) = \hat{\varphi}_{in}(f(x^-))$

(***Dynamical Casimir Effect***)

Moving Mirror Model in 1+1 dim. mimics 3+1 dim. spherical gravitational collapse.

$$x^{+} = f(x^{-}) = -\frac{1}{\kappa}\ln\left(1+e^{-\kappa x^{-}}\right)$$

$$f(x^{-} \approx -\infty) \approx x^{-}$$ *The mirror does not move in the past.*

$$f(x^{-} \approx \infty) \approx -\frac{1}{\kappa}\exp\left(-\kappa x^{-}\right)$$

The mirror accelerates and approaches the light trajectory, $x^{+} = 0.$

acceleration

The mirror emits thermal flux in the late time.

$$\boxed{\left\langle 0_{in} \middle| \hat{T}_{--}(x^- >> 1/\kappa) \middle| 0_{in} \right\rangle = \frac{\pi}{12} T^2}$$

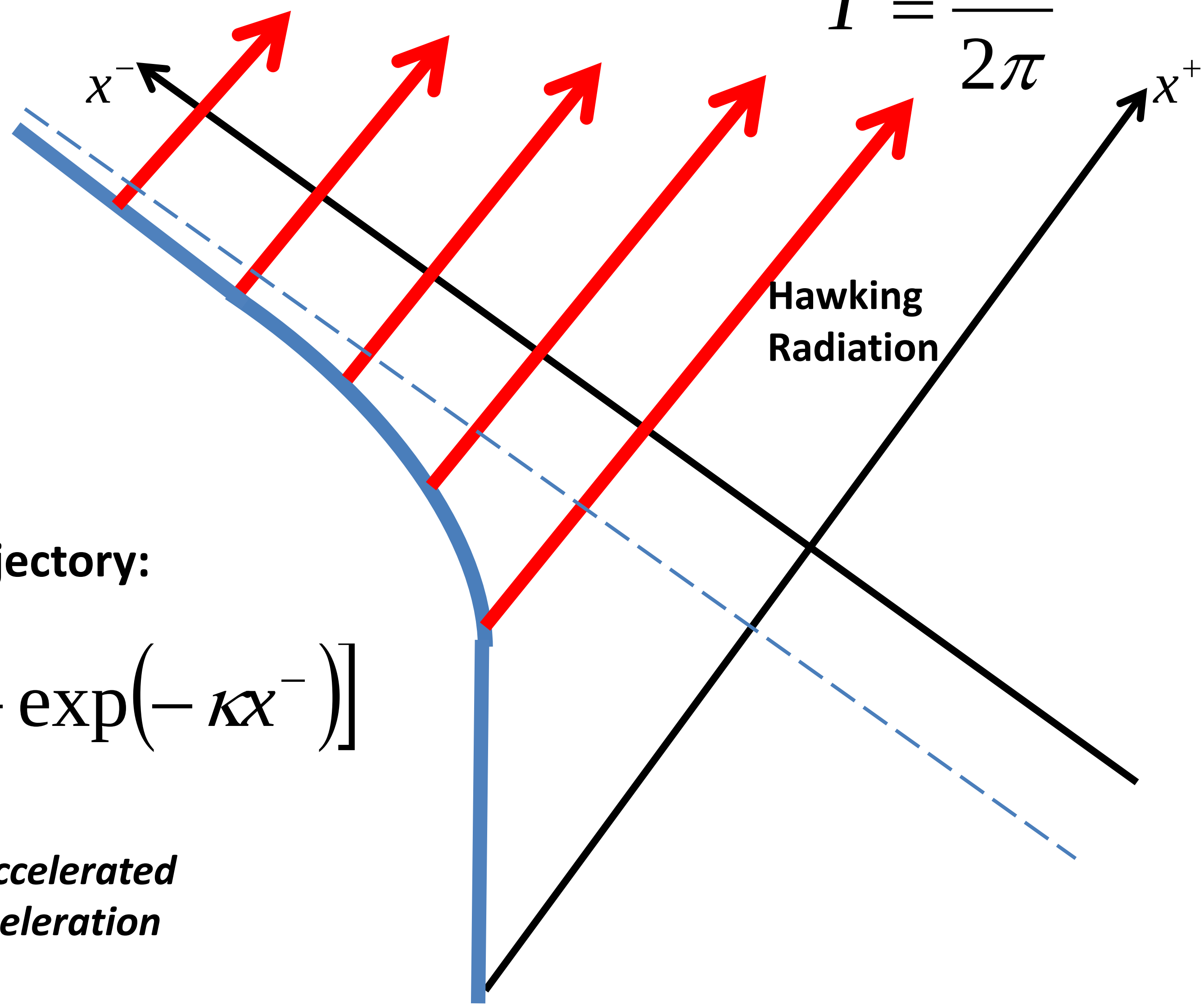
Temperature: $T = \dfrac{\kappa}{2\pi}$ $\leftarrow$ ***acceleration***

$$\left\langle 0_{in} \middle| \hat{a}^{(out)}{}_{\omega}{}^{\dagger} \hat{a}^{(out)}{}_{\omega} \middle| 0_{in} \right\rangle \propto \frac{1}{\exp\left(\dfrac{2\pi}{\kappa}\omega\right) - 1}$$

Hawking Radiation!

1+1 dim Moving Mirror Model as analogue of Hawking radiation emission

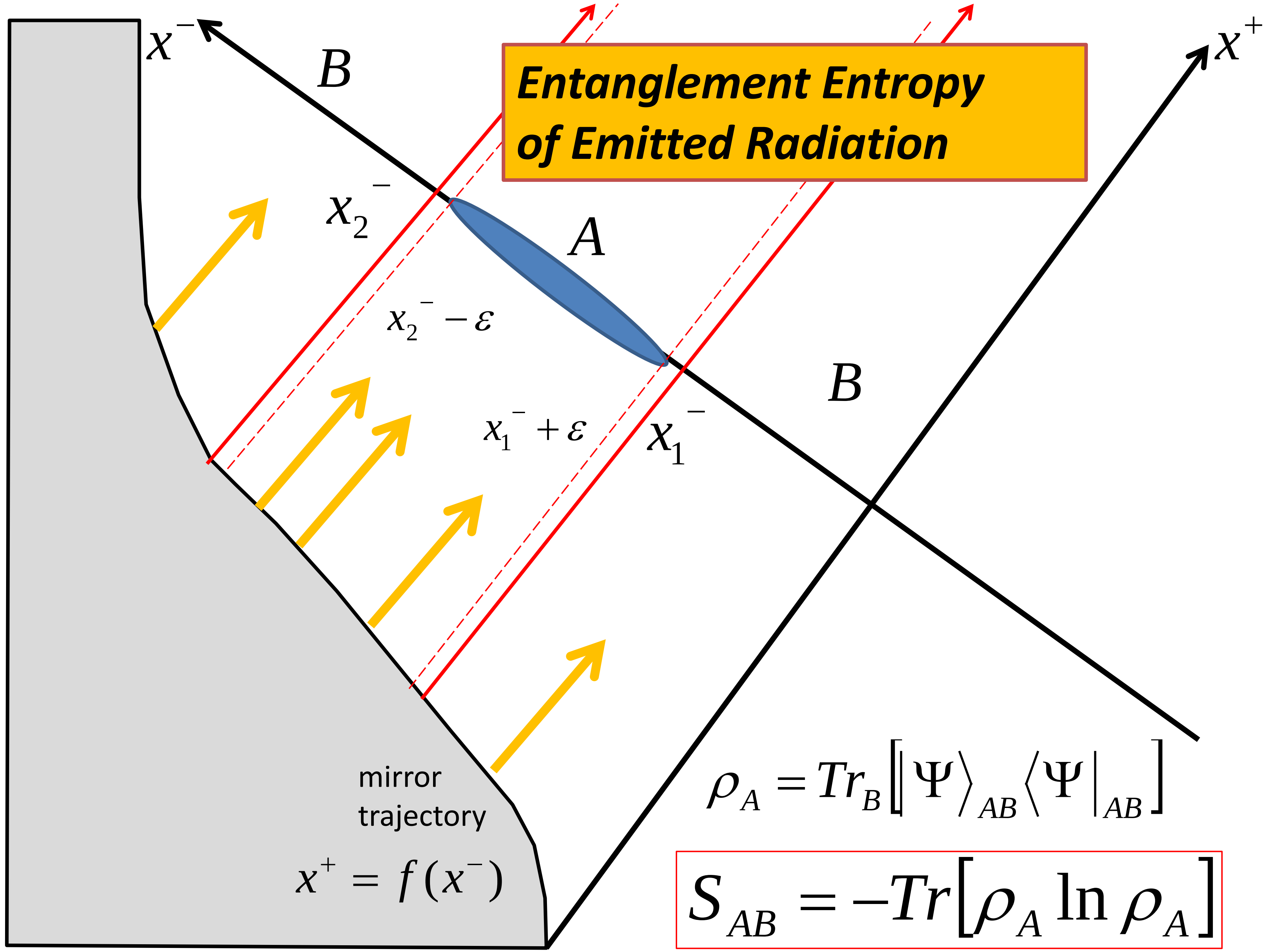
$$T = \frac{\kappa}{2\pi}$$



Mirror Trajectory:

$$x^+ = -\ln\left[1 + \exp\left(-\kappa x^-\right)\right]$$

At rest
→ Gradually accelerated
→ Uniform acceleration



Entanglement Entropy of Emitted Radiation
x^-
x^+
B
A
B
x_2^-
$x_2^- - \varepsilon$
$x_1^- + \varepsilon$
x_1^-
mirror trajectory
$x^+ = f(x^-)$
$\rho_A = Tr_B\left[|\Psi\rangle_{AB}\langle\Psi|_{AB}\right]$
$S_{AB} = -Tr\left[\rho_A \ln \rho_A\right]$

Entanglement entropy:

$$S_{AB} = \frac{1}{12}\ln\left[\frac{\left(f(x_2^{\;-}) - f(x_1^{\;-})\right)^2}{\varepsilon^2 \partial_- f(x_2^{\;-}) \partial_- f(x_1^{\;-})}\right]$$

Lattice spacing (UV cutoff)

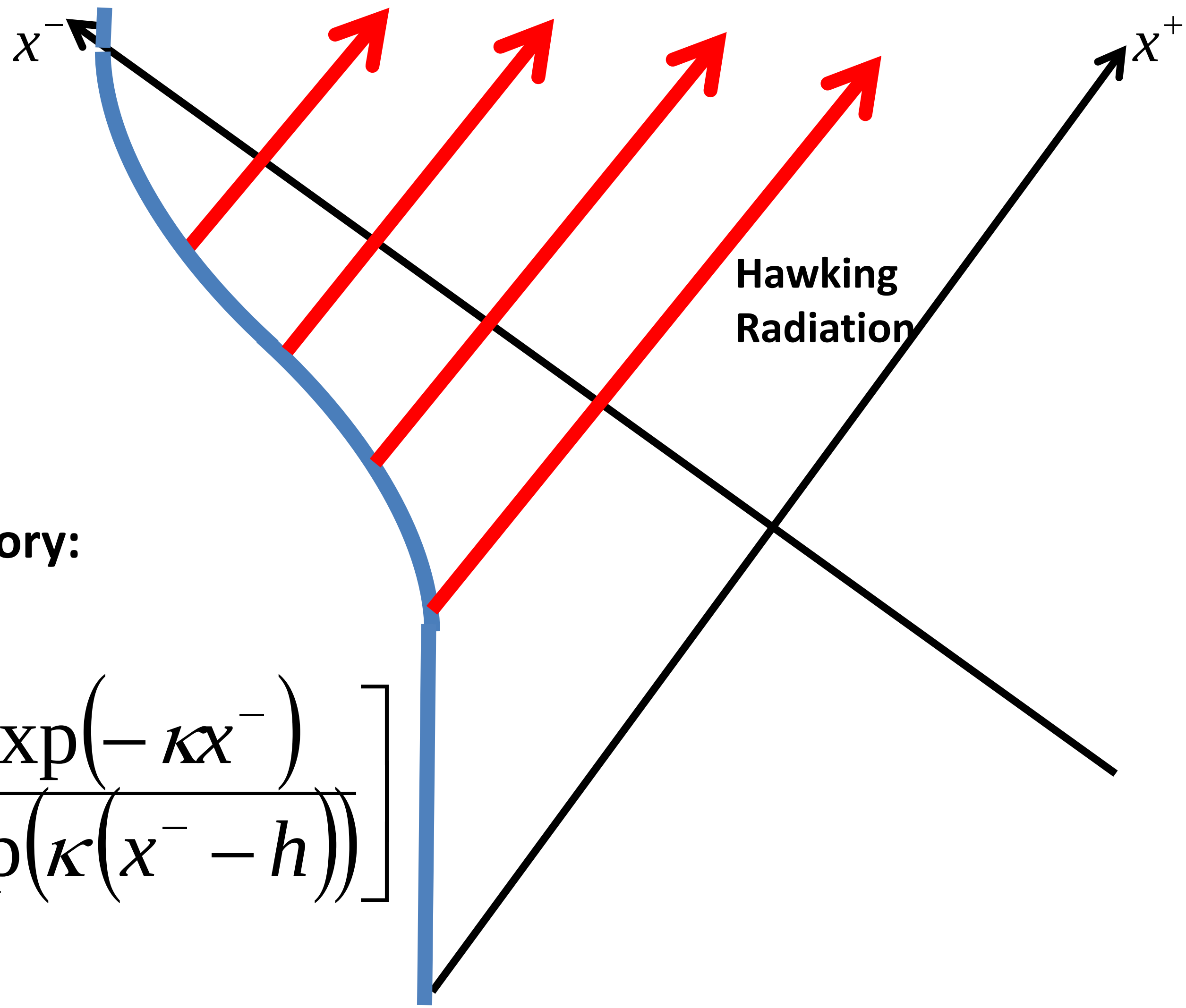
Renormalized entanglement entropy:

$$S_{ren} = S_{AB} - S^{(vac)}{}_{AB}$$

$$= \frac{1}{12}\ln\left[\frac{\left(f(x_2^{\;-}) - f(x_1^{\;-})\right)^2}{\partial_- f(x_2^{\;-}) \partial_- f(x_1^{\;-}) \left(x_2^{\;-} - x_1^{\;-}\right)^2}\right]$$

Holzhey-Larsen-Wilczek

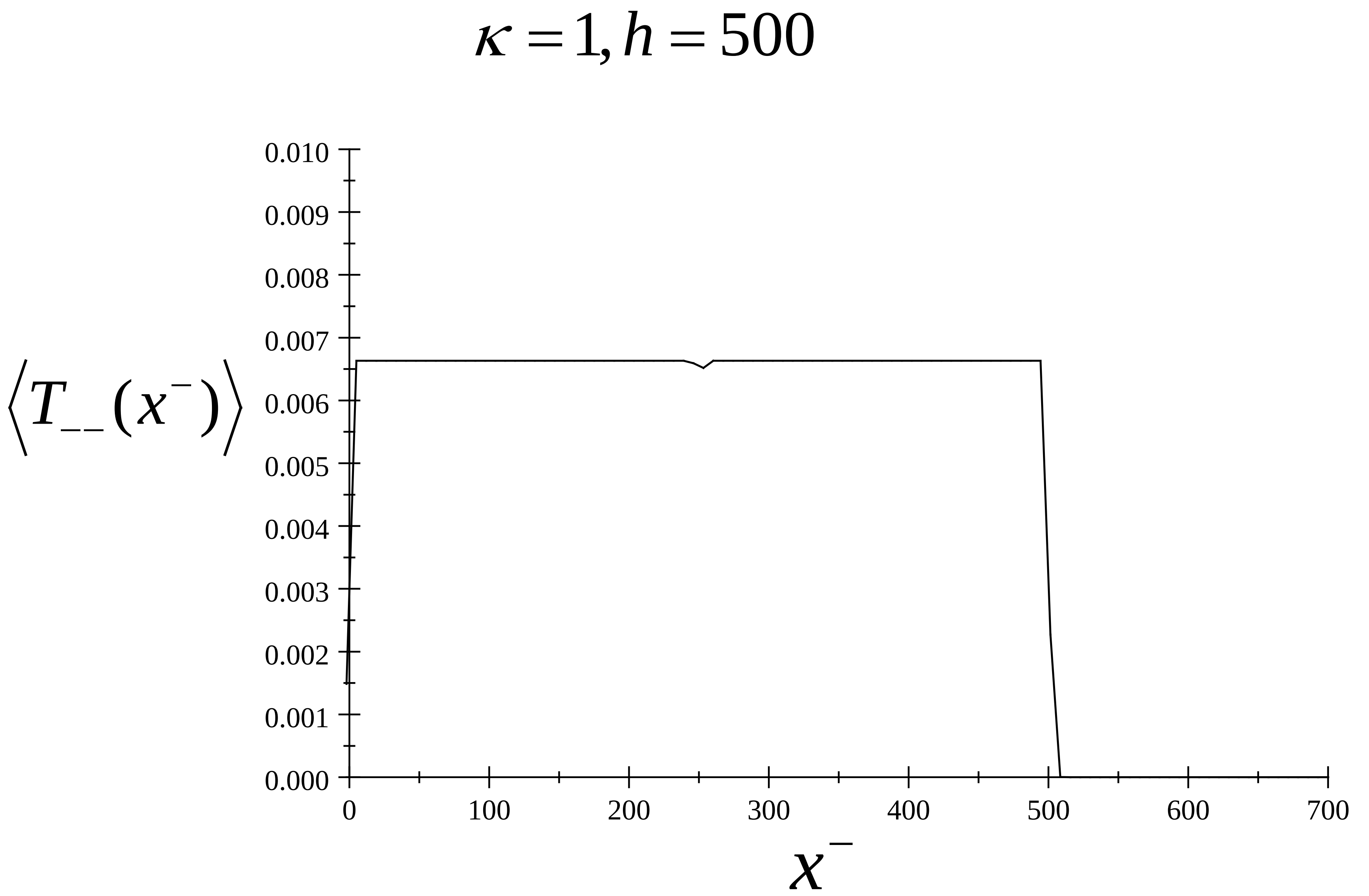
BH Evaporation Trajectory



Mirror Trajectory:

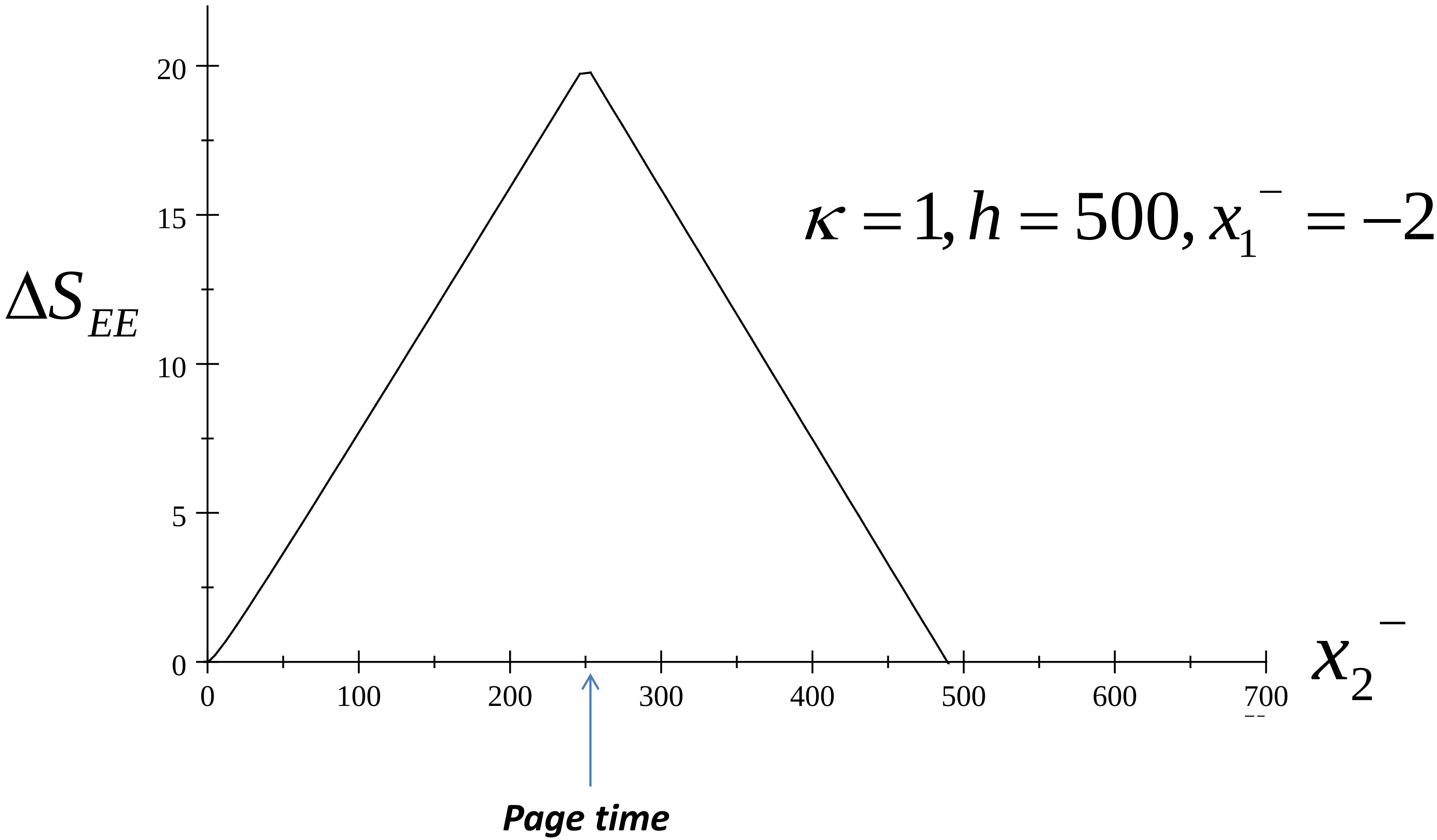
$$x^+ = -\ln\left[\frac{1+\exp\left(-\kappa x^-\right)}{1+\exp\left(\kappa\left(x^- - h\right)\right)}\right]$$

At rest → Uniform Acceleration → At rest



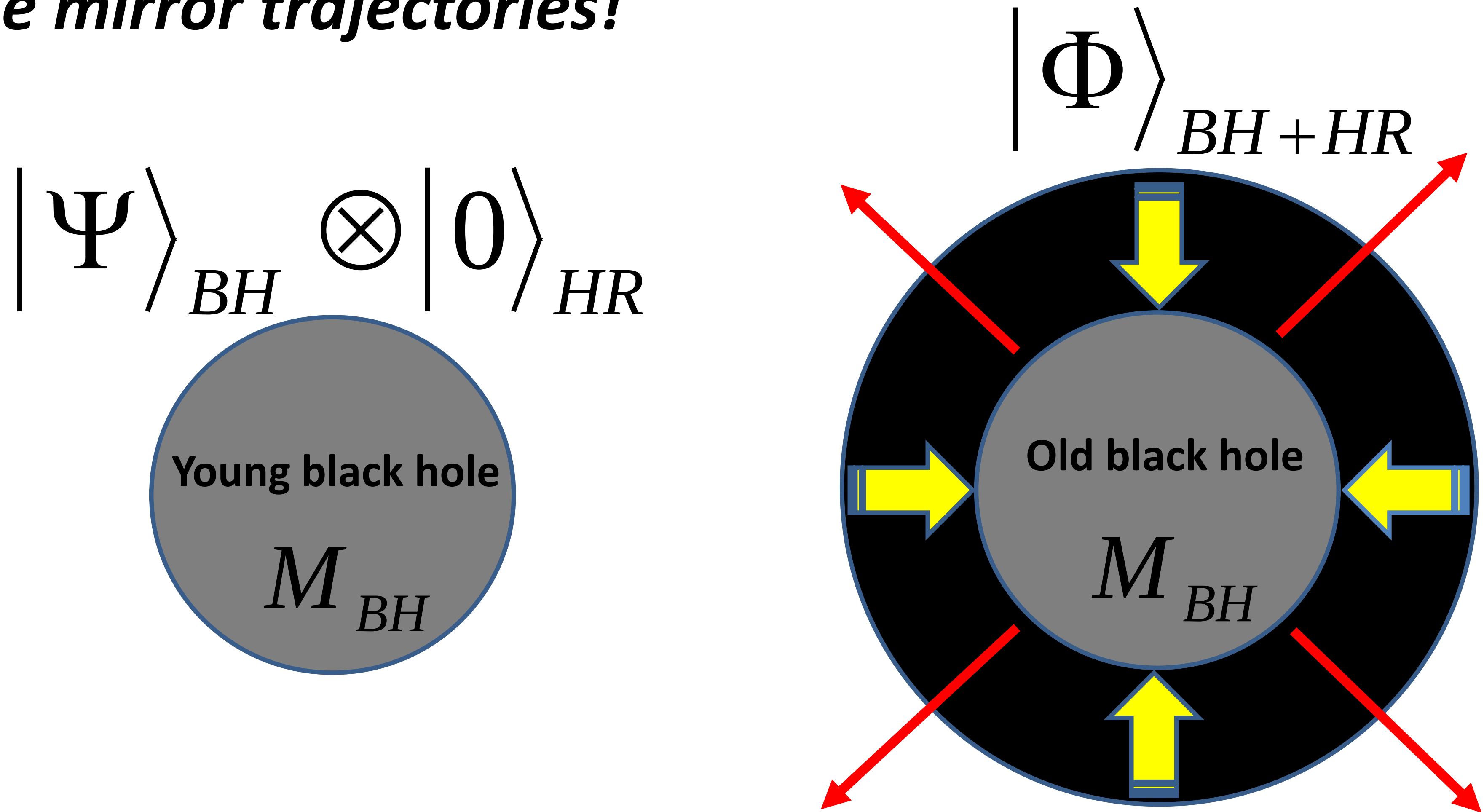
$\kappa = 1, h = 500$
$\langle T_{--}(x^-) \rangle$
0.010
0.009
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0.000
0
100
200
300
400
500
600
700
x^-

Page Curve for BH Evaporation Trajectory



Thanks to Daniel Harlow

In order to reproduce the Page curve, very strange time evolution induced by nonlocality is required for the mirror trajectories!

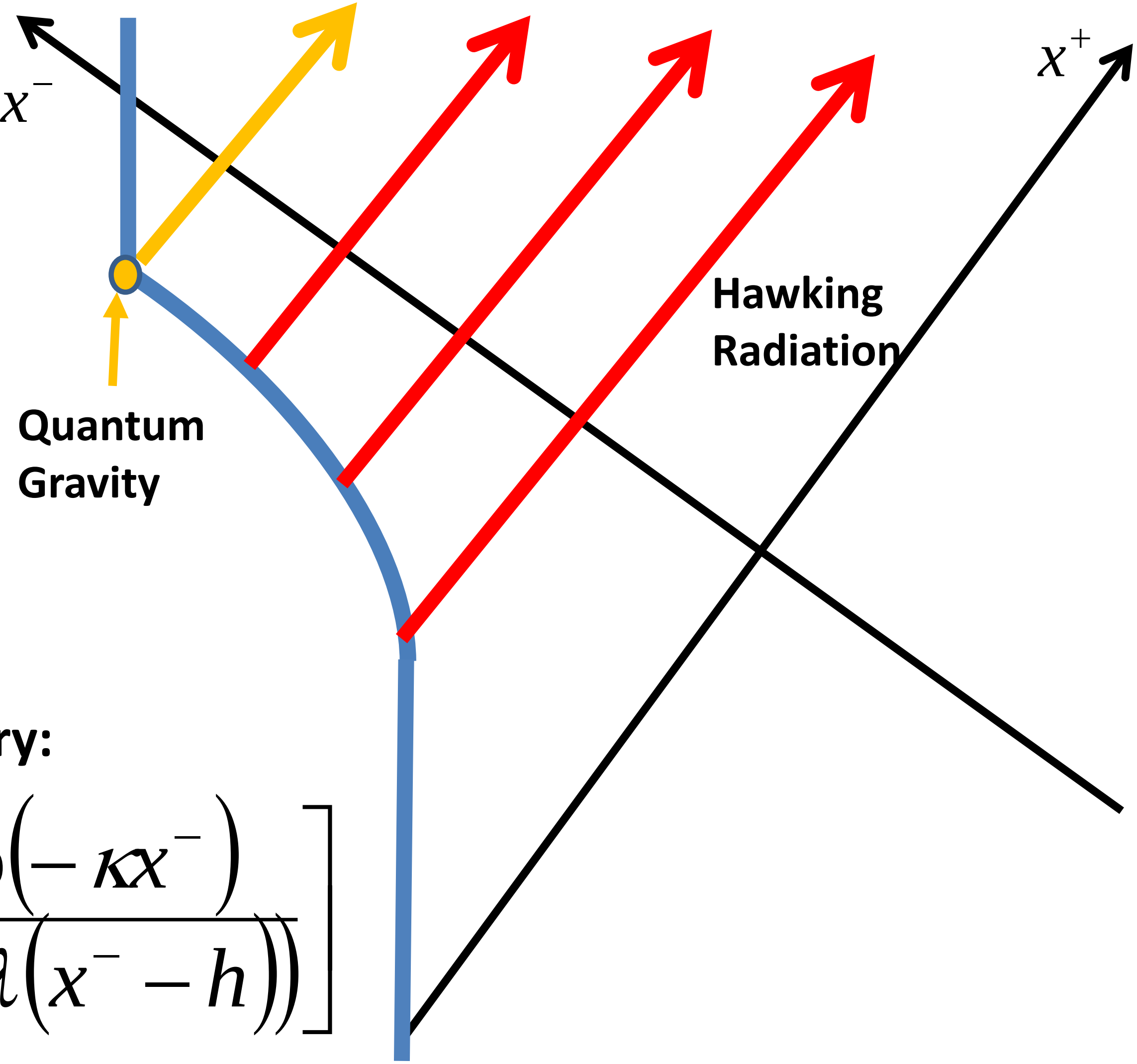


Quite different time schedules of information leakage for black holes with the same mass.

Possible modification of the Page curve, assuming local dynamics.

$$\lambda >> \kappa$$

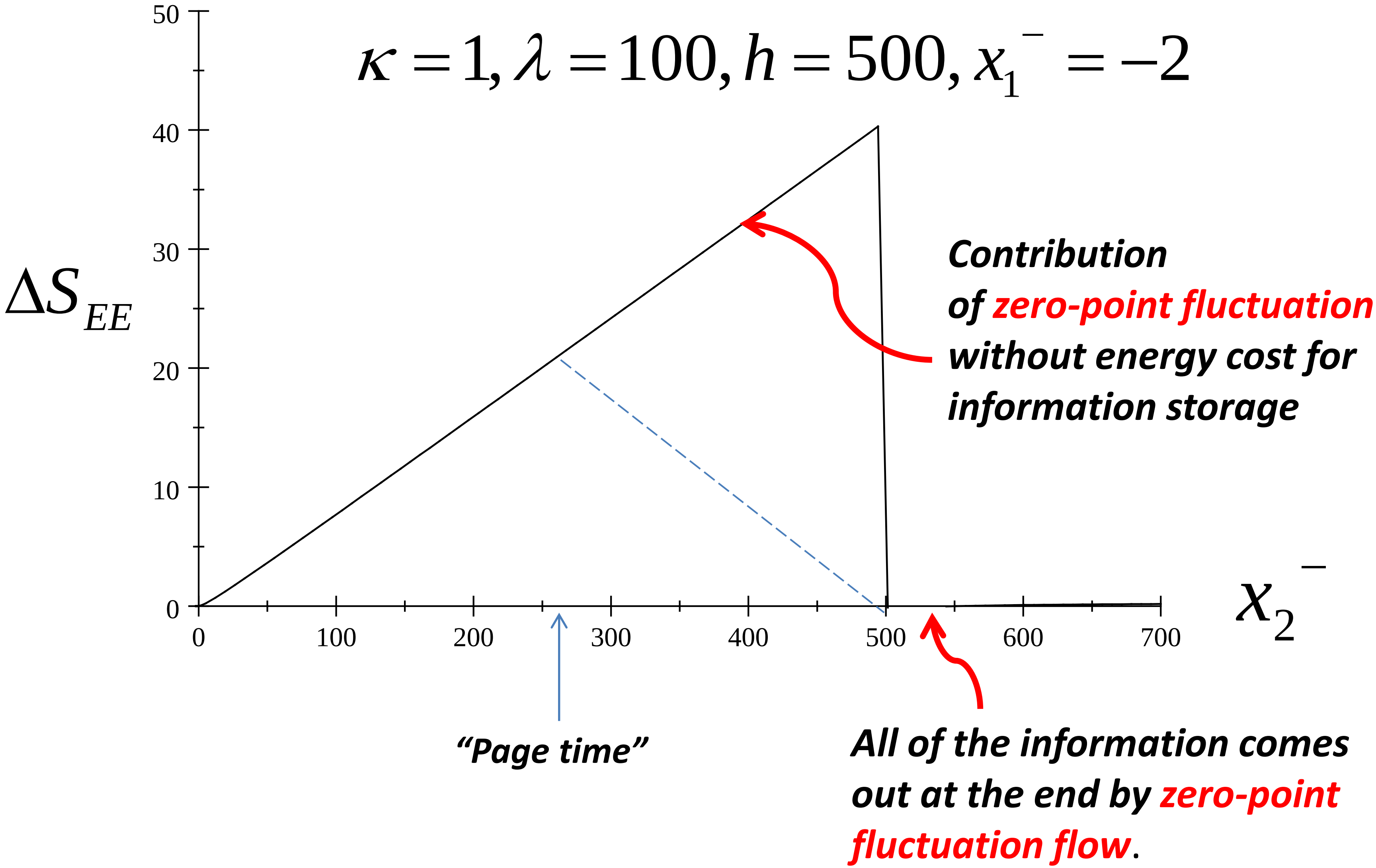
Planck energy scale — *BH scale*



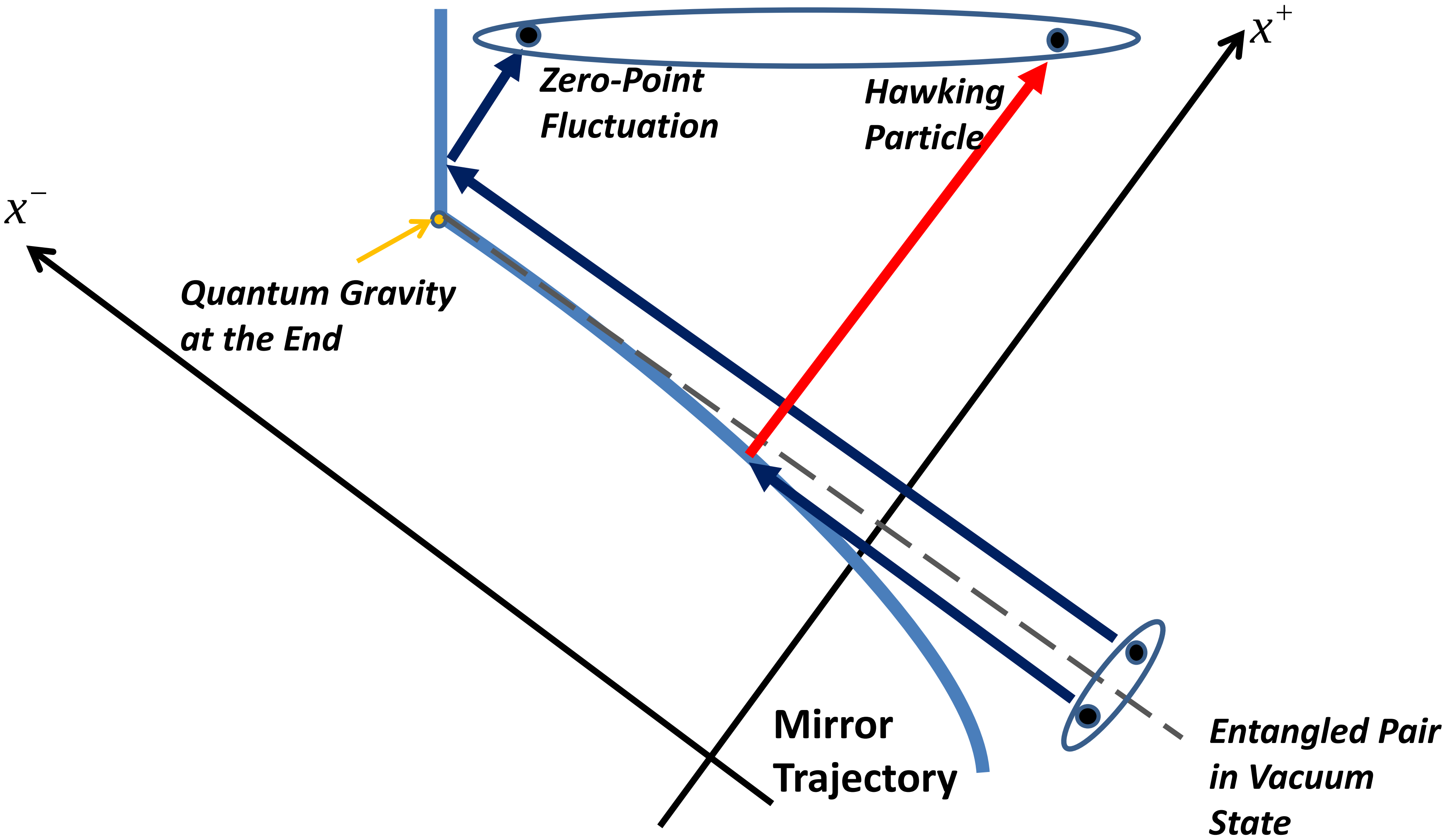
Mirror Trajectory:

$$x^+ = -\ln\left[\frac{1+\exp(-\kappa x^-)}{1+\exp(\lambda(x^- - h))}\right]$$

Modified Page Curve for BH Evaporation



Information Retrieval without Energy at the End



The entangled partner of the Hawking particle is zero-point fluctuation with zero energy. ***(Wilczek, Hotta-Schützhold-Unruh)***

Entangled Partner

Particle A ***Particle B***

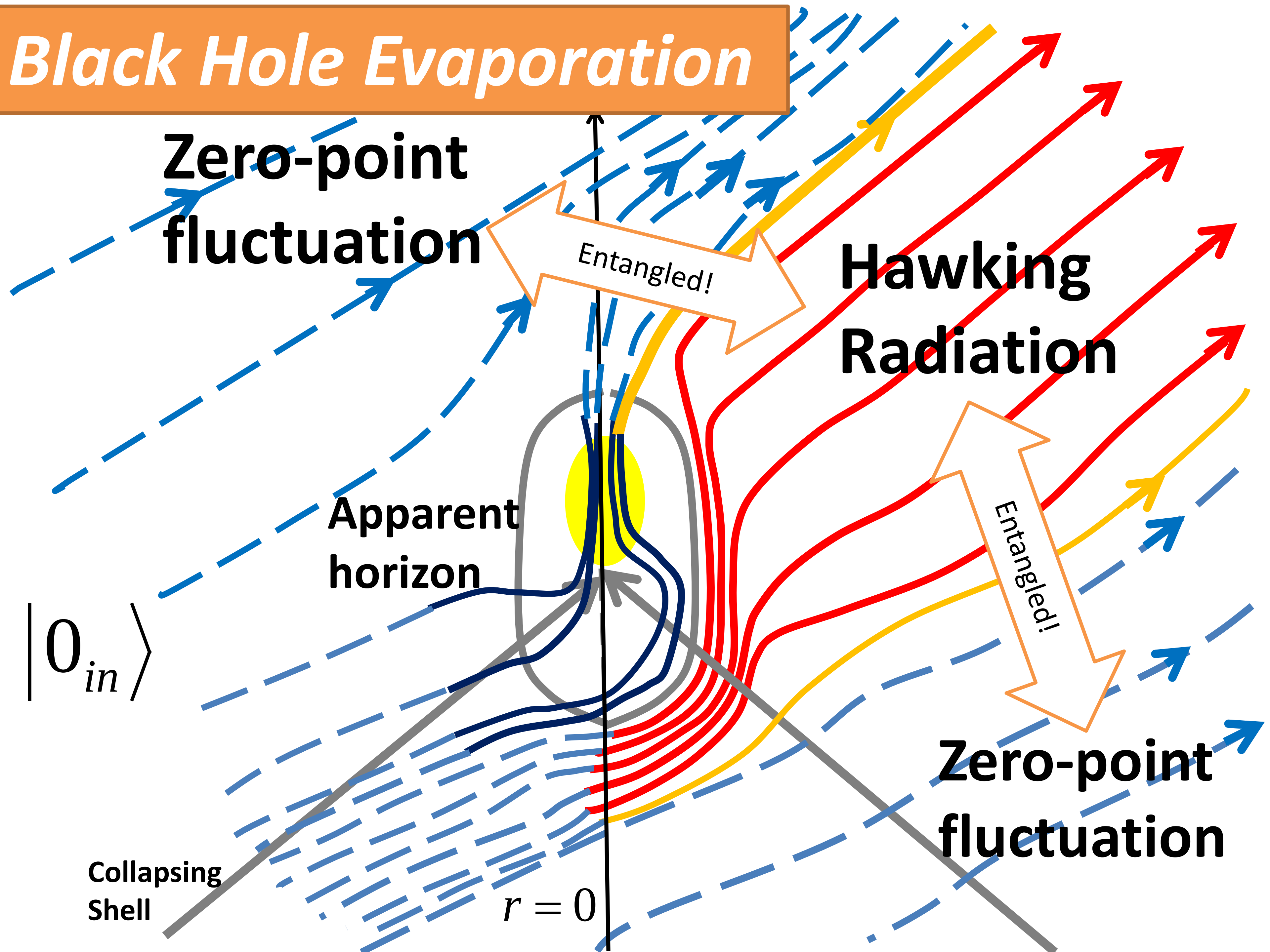
Entanglement

Hawking Particle

Zero-Point Fluctuation Flow with Zero Energy

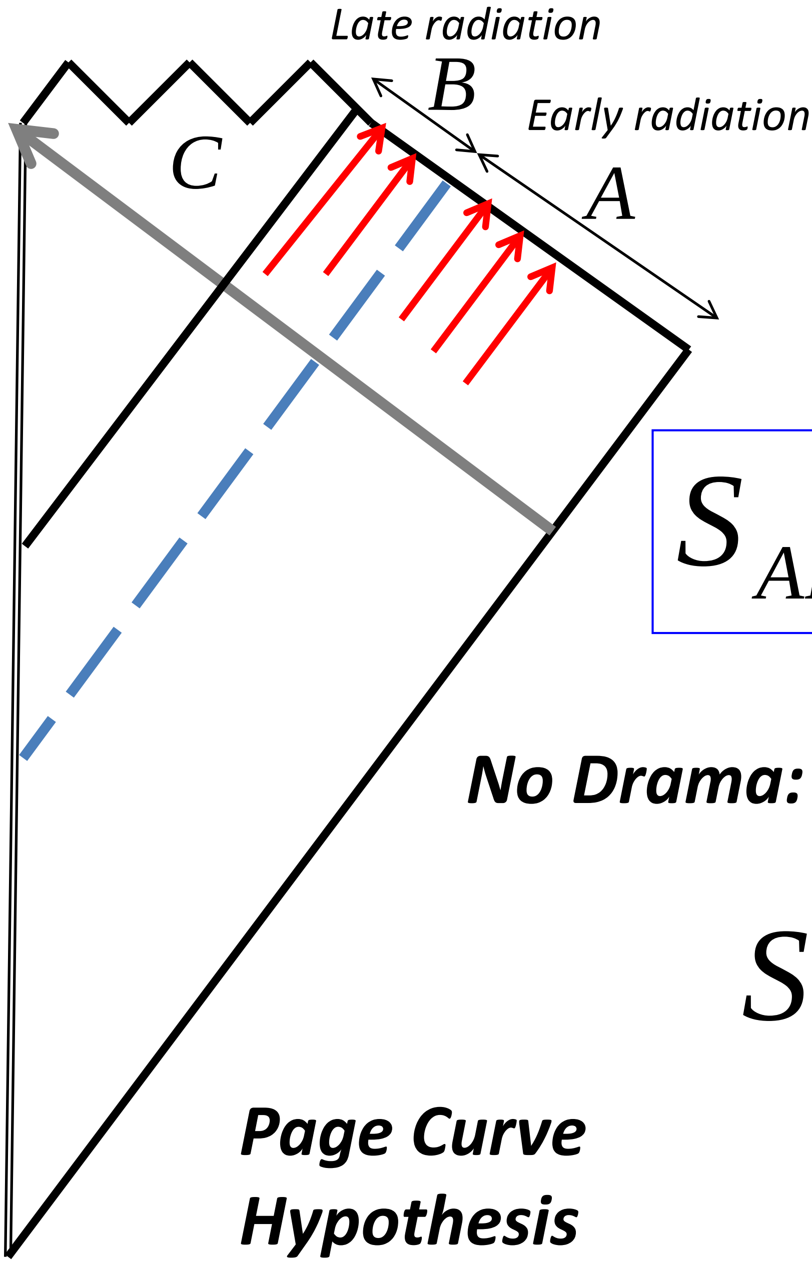
(Wilczek, Hotta-Schützhold-Unruh, cf. Hawking-Perry-Strominger)

Therefore,
*the information loss problem may **not** be so serious, because small (or zero) amount of energy is enough to carry huge amount of quantum information in principle.*



Black Hole Evaporation
Zero-point fluctuation
Entangled!
Hawking Radiation
Apparent horizon
$|0_{in}\rangle$
Entangled!
Zero-point fluctuation
Collapsing Shell
$r = 0$

Strong Subadditivity "Paradox"



Strong subadditivy:

$$\boxed{S_{AB} \geq S_B + S_{ABC} - S_{BC}}$$

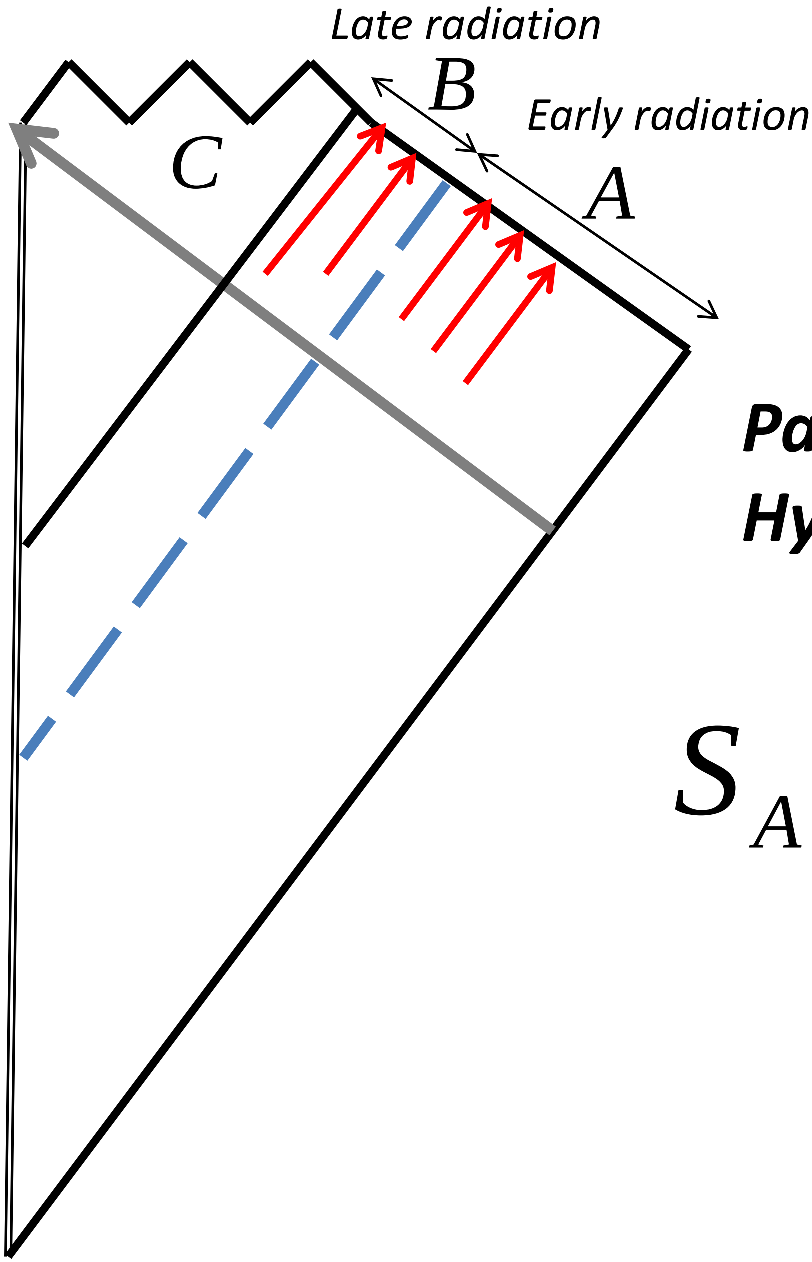
No Drama: $S_{BC} = 0,\ S_{ABC} = S_A$

$$S_{AB} \geq S_B + S_A$$

Page Curve Hypothesis

$$S_A > S_{AB}$$

Strong Subadditivity "Paradox"

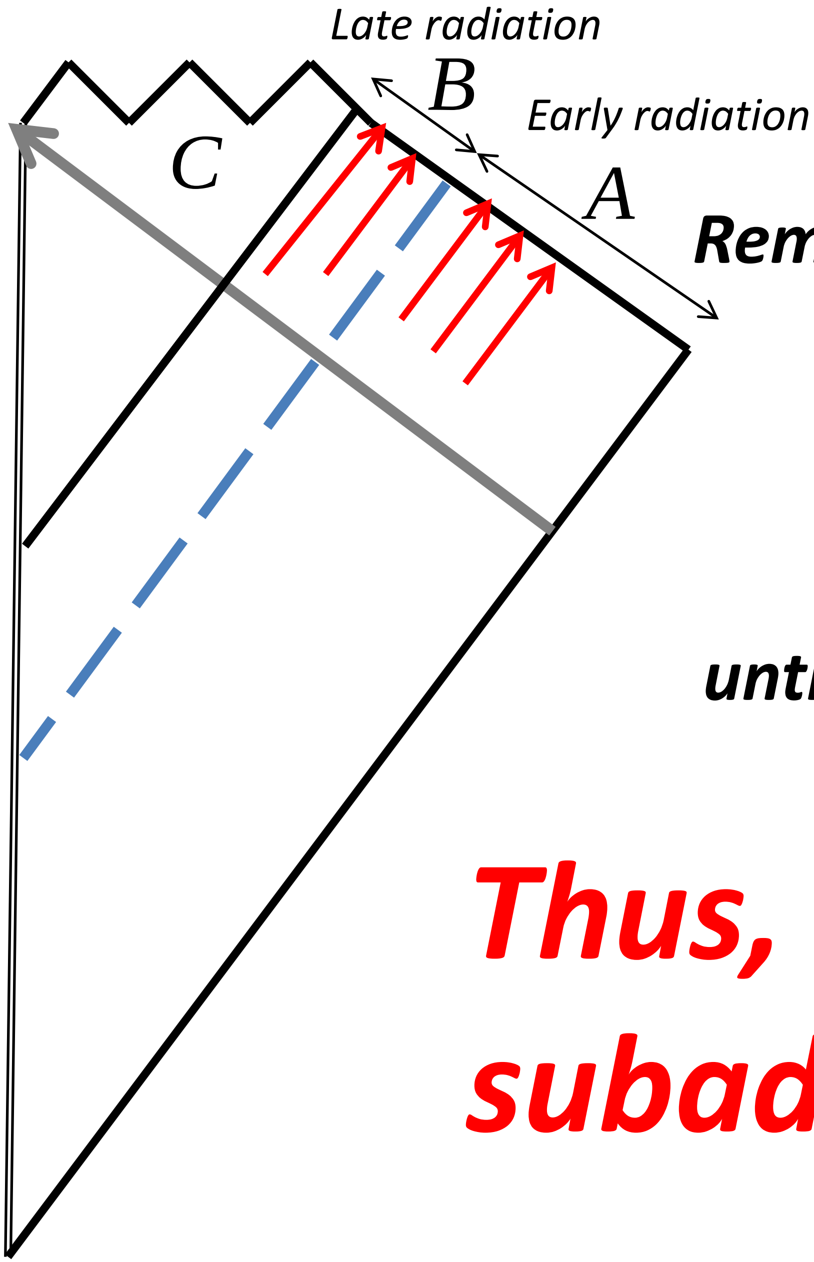


Page Curve Hypothesis $S_A > S_{AB}$

$$S_A > S_{AB} \geq S_B + S_A$$

$$\boxed{0 > S_B}$$

Strong Subadditivity “Paradox”



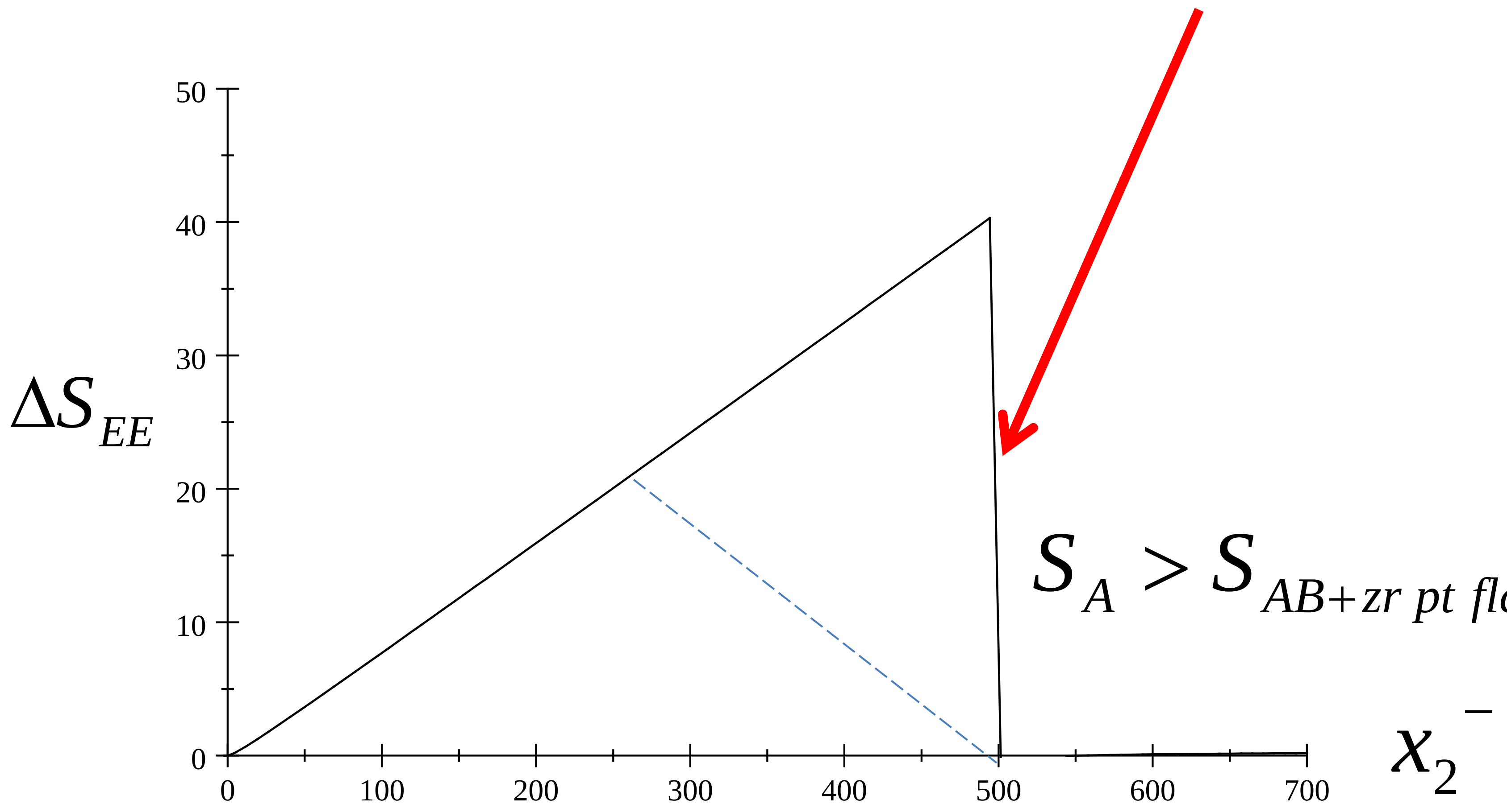
Remnant& Zero-Point Fluctuation Flow

$$S_A < S_{AB}$$

until the last burst.

Thus, no strong subadditivity paradox!

We don't care the no drama condition breaks at the last burst,
because the horizon is affected by quantum gravity.



The last burst

Summary

○ *Adopting canonical typicality for nondegenerate systems with nonvanishing Hamiltonians, the entanglement becomes non-maximal, and BH firewalls do not emerge.*

○ *Typical states must be Gibbs states for smaller quantum systems. If we have stable Gibbs states for old Schwarzschild BH's (and small AdS BH's), the heat capacity must be positive. Because it is actually negative, the states of BH evaporation are not typical.*

$\Rightarrow$ *Inevitable Modification of the Page Curve*

Note: for a large AdS BH and Hawking radiation in a thermal equilibrium, the entanglement entropy equals the thermal entropy of the smaller system.

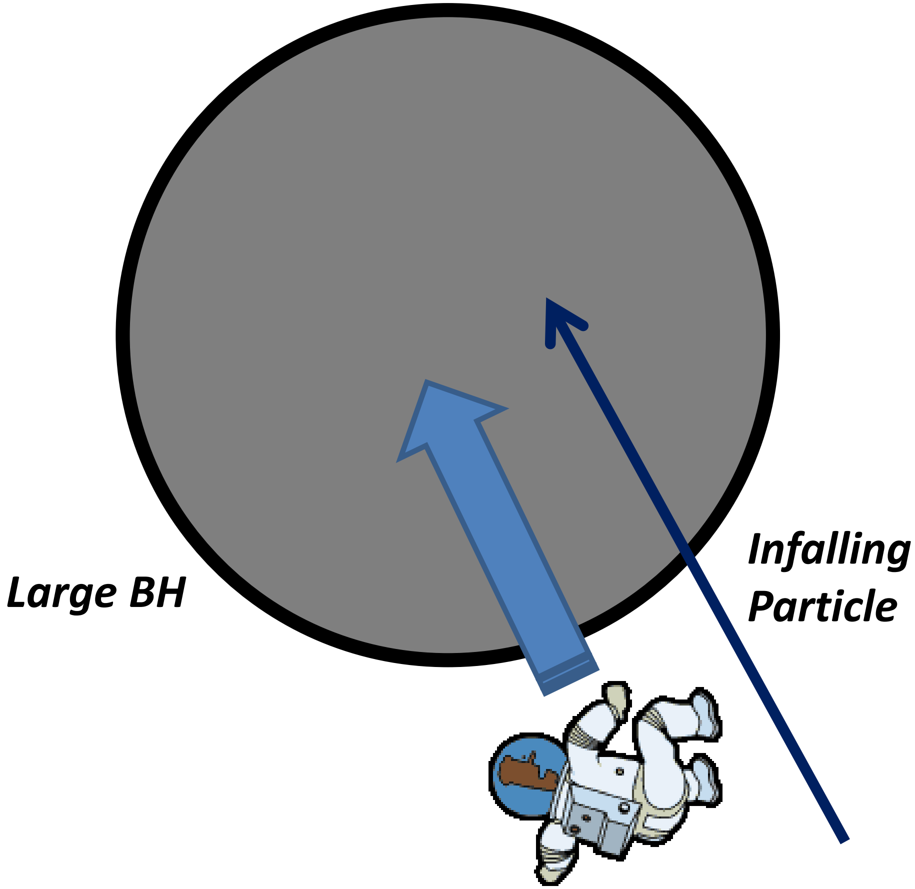
IV. Quantum Measurement of BH Firewalls

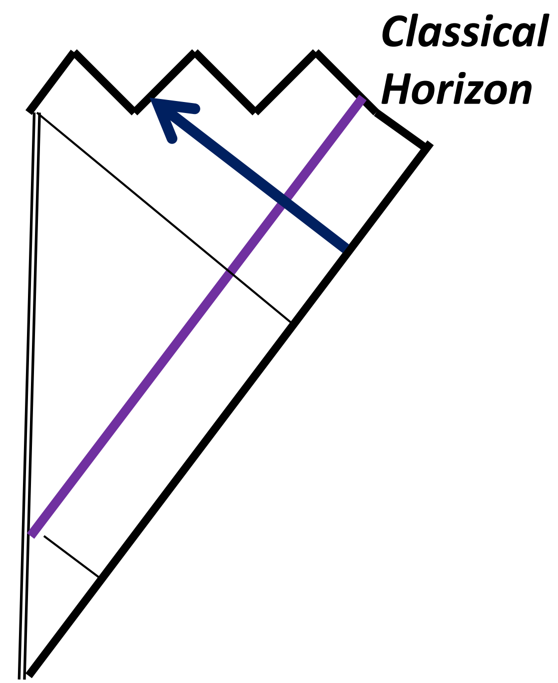
"Another Firewall Paradox"

M. Hotta, J. Matsumoto and K. Funo, Phys. Rev. D89, 124023 (2014)

The strong subadditivity paradox has been resolved.

Free-fall observers do not encounter firewalls when come across event horizon *in an average meaning.*





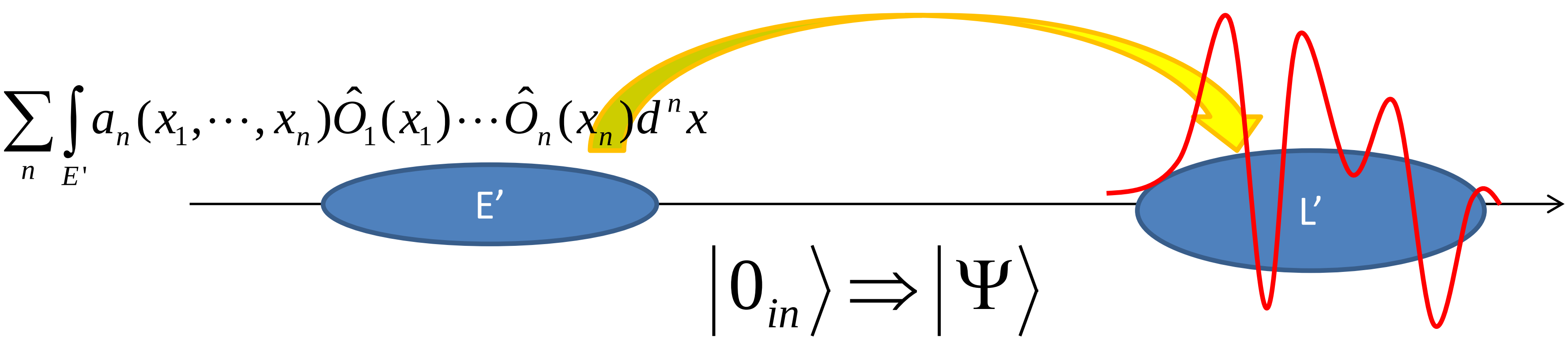
However, we have another possibility of firewall emergence.

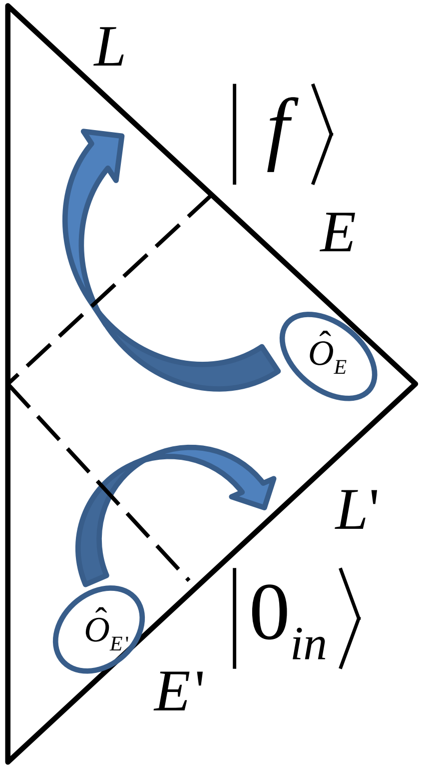
The point is Reeh-Schlieder theorem in quantum field theory.

Reeh-Schlieder theorem:

The set of states generated from $|0_{in}\rangle$ by the polynomial algebra of local operators in any bounded spacetime region is dense in the total Hilbert space of the field. Thus, in principle, any state can be arbitrarily closely reproduced by acting a polynomial of local operators of E' on $|0_{in}\rangle$.

$$\forall |\Psi\rangle \approx \sum_n \int_{E'} a_n(x_1,\cdots,x_n)\hat{O}_1(x_1)\cdots\hat{O}_n(x_n)d^n x|0_{in}\rangle$$



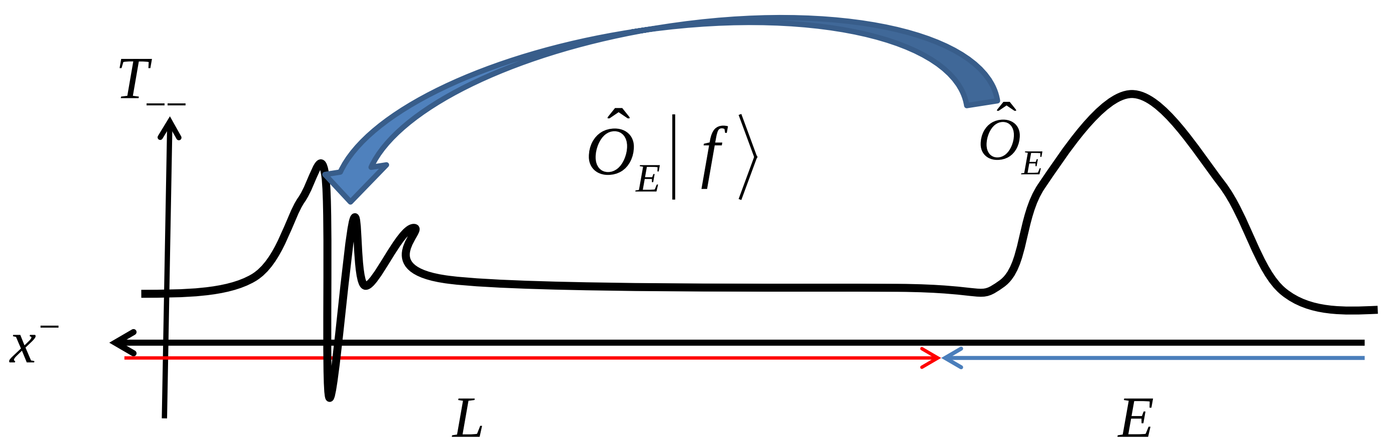


Note that the Reeh-Schlieder property is maintained in the time evolution:

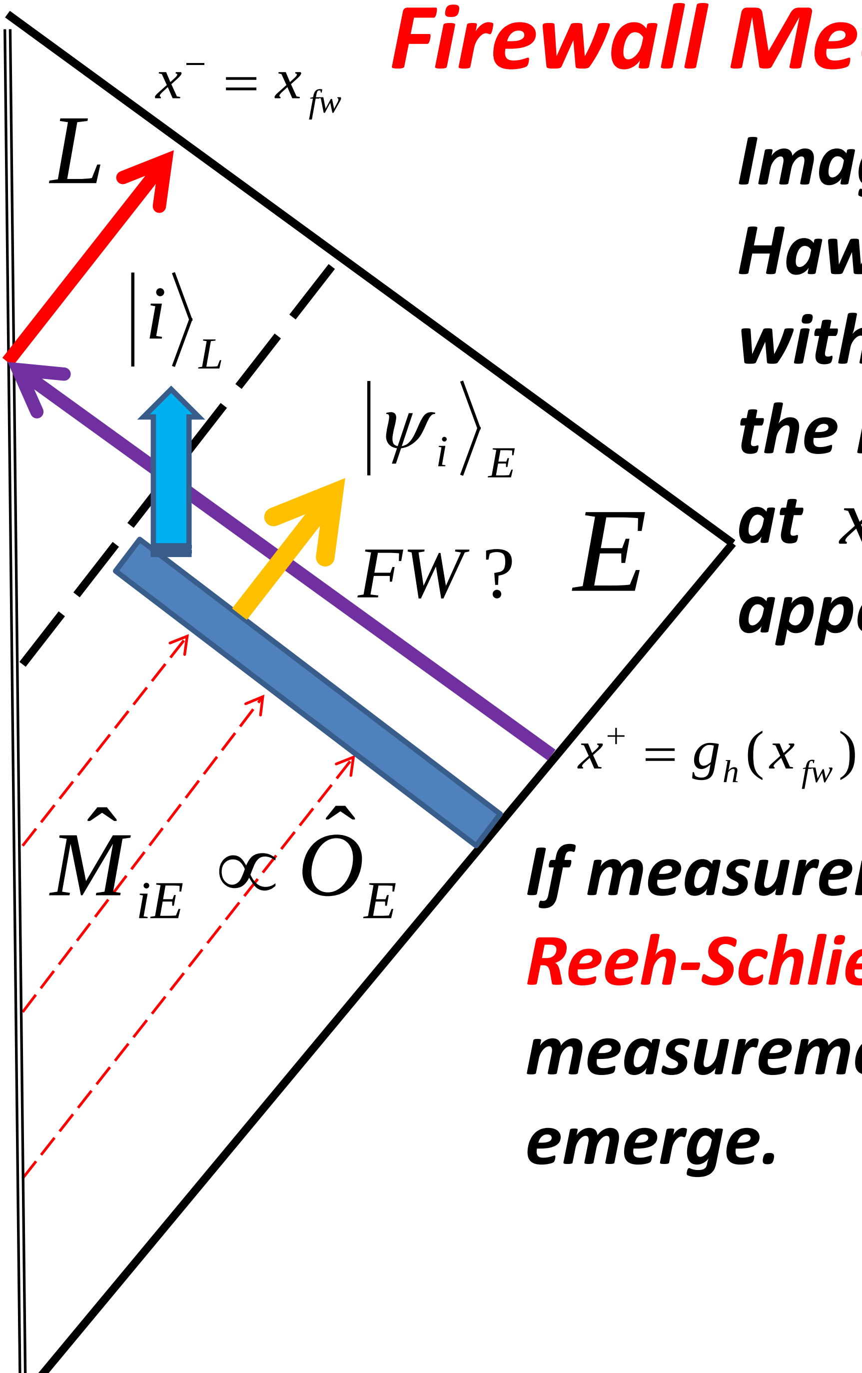
$$|0_{in}\rangle \to |f\rangle.$$

Even in the future infinity,
we may remotely generate any excitation ***with some probability smaller than 1.***

$$\hat{O}_E = \sum_n \int_E a_n(x_1,\cdots,x_n)\hat{O}_1(x_1)\cdots\hat{O}_n(x_n)d^n x$$



Firewall Measurement Paradox:



Imagine that, besides the background Hawking radiation, a wave packet with positive energy of the order of the radiation temperature appears at $x^- = x_{fw}$. Then the firewall (FW) appears at $x^+ = g_h(x_{fw})$.

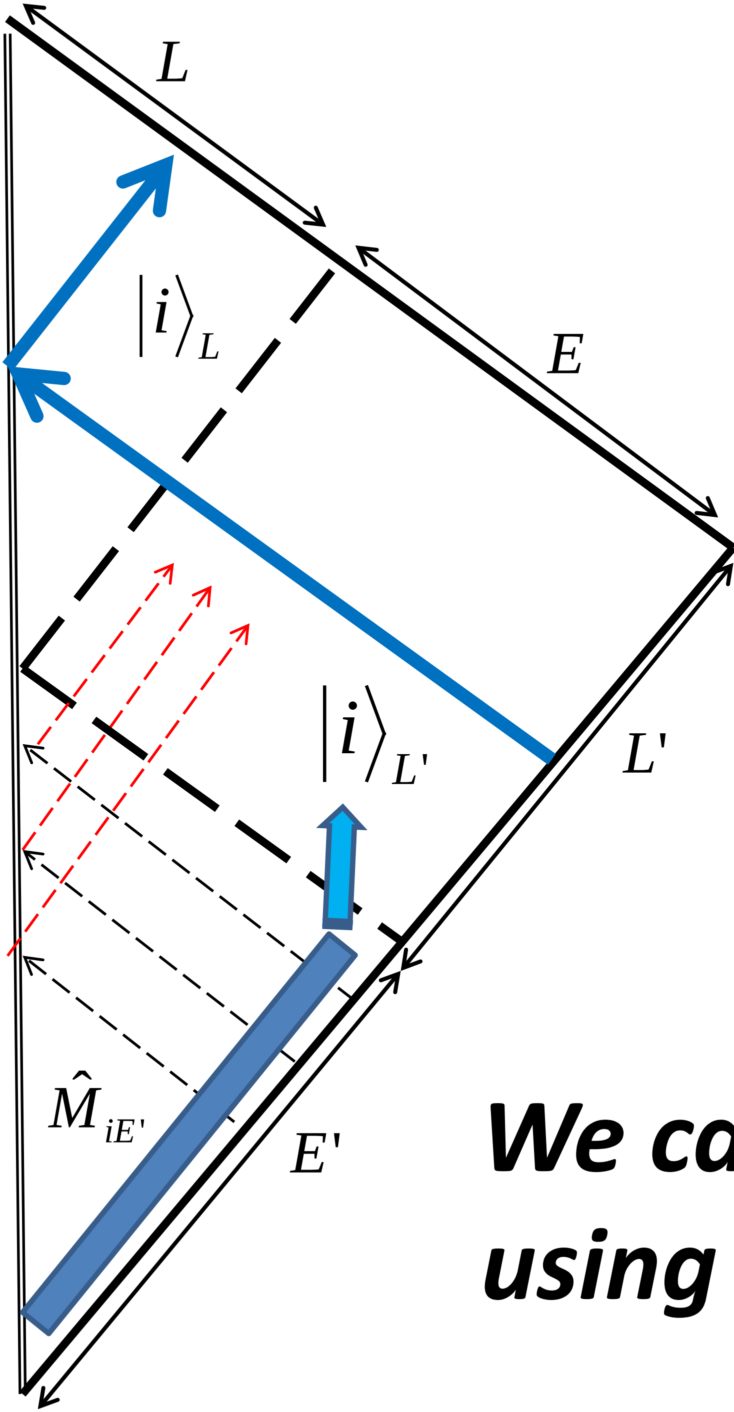
If measurement operator is constructed from Reeh-Schlieder operation, an arbitrary post-measurement state including firewalls can emerge.

$$|f\rangle = \sum_i \underbrace{|\psi_i\rangle_E}_{\text{Measured}} \underbrace{|i\rangle_L}_{\text{Firewall!}}$$

V. Informational Cosmic Censorship Conjecture

"Resolution of the Paradox
from a viewpoint of
Quantum Measurement Energy Cost"

M. Hotta, J. Matsumoto and K. Funo, Phys. Rev. D89, 124023 (2014)



Because the mirror merely stretches the modes of the field, the future measurement is equivalent to a past measurement for the in-vacuum state.

$$\hat{M}_{iE} \Leftrightarrow \hat{M}_{iE'}$$

We can analyze the problem using past infinity.

The local measurements generally inject energy on average to the system in $|0_{in}\rangle$ owing to its passivity property (Pusz and Woronowicz). Thus the measurements always require an energy cost.

Though the Reeh-Schlieder theorem is mathematically correct, it does not guarantee that the measurement energy to create $|i_L\rangle$ is finite.

If $\hat{M}^{\dagger}{}_{iE'}\hat{M}_{iE'}$ is regular,
no outstanding peak of energy flux appears.

The two-point correlation functions for non-singular measurements simply obey a power-law decay as a function of the distance. ⇒ No Firewalls!

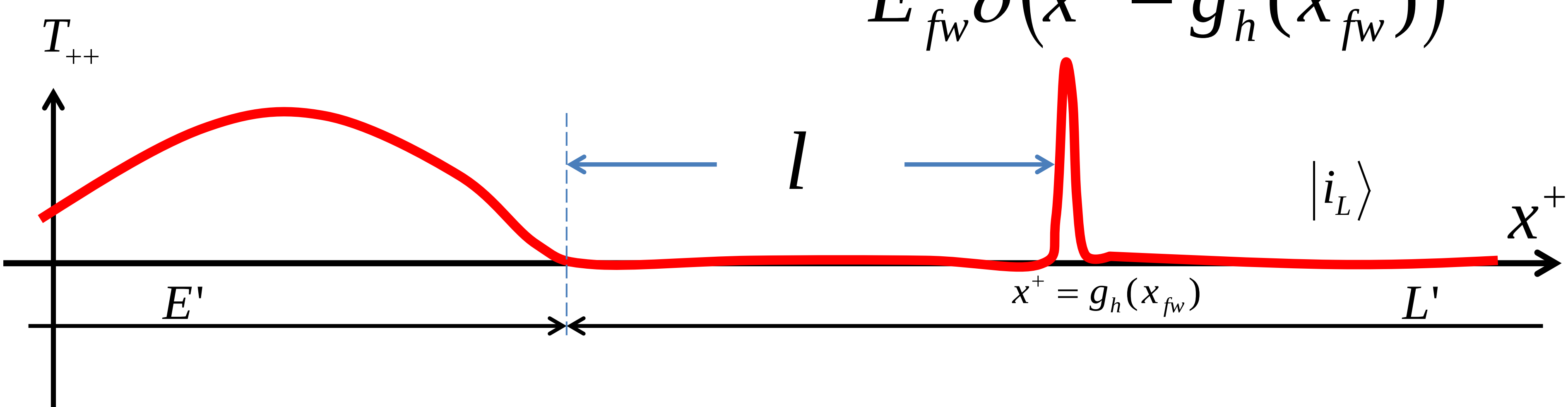
Hotta, Matsumoto and Funo, Phys. Rev. D89, 124023 (2014).

If we assume FW appears at $x^+ = g_h(x_{fw})$ ***with less-than-1 probability in a finite-energy measurement, then***

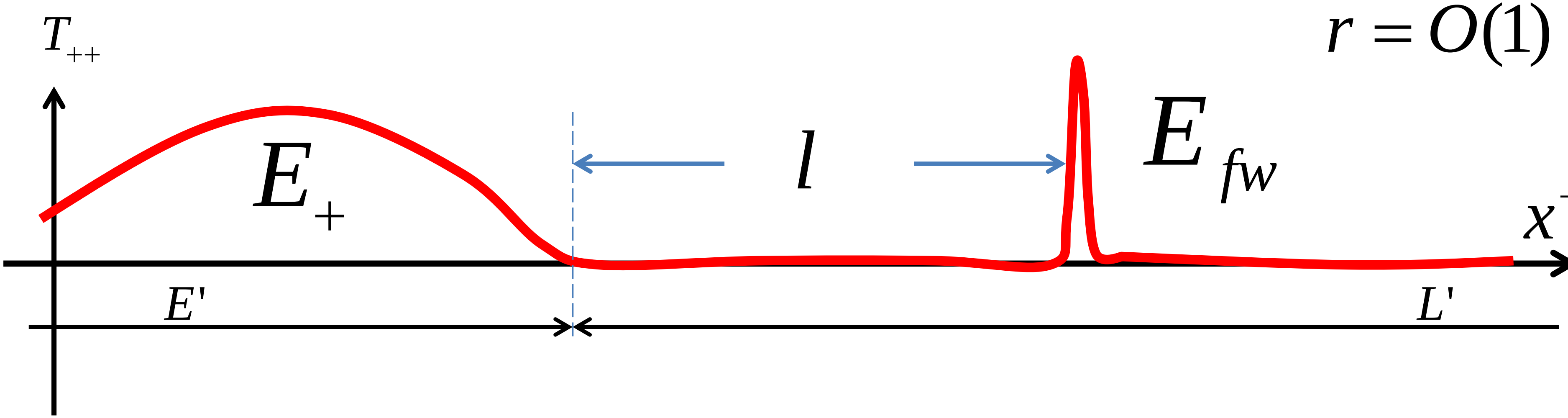
$$E_{fw} < O\left(\frac{1}{12\pi l}\right) << E_{planck}$$

⇒No Firewall appears!

$$E_{fw}\delta\left(x^+ - g_h(x_{fw})\right)$$



Hotta, Matsumoto and Funo, Phys. Rev. D89, 124023 (2014).

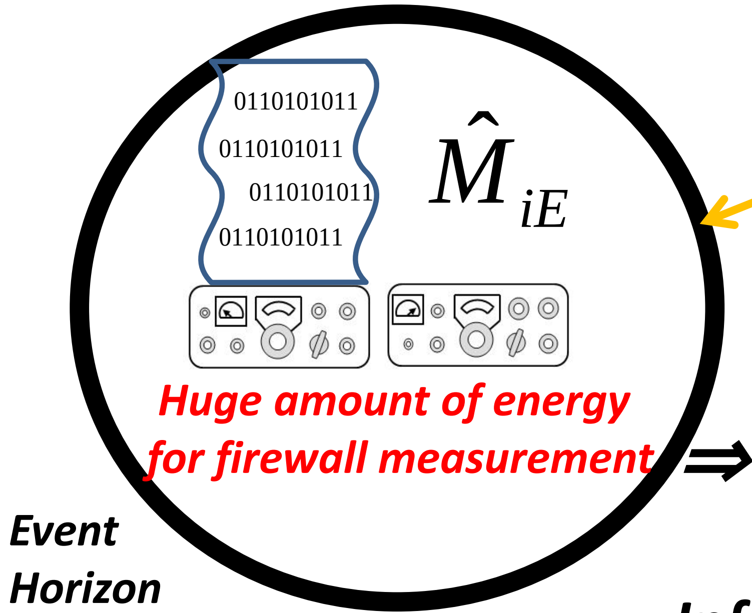


Energy cost of measurement: $$E_+ \geq \frac{rE_{fw}}{1-12\pi rlE_{fw}}$$

$$E_{fw} \to \frac{1}{12\pi rl} \qquad E_+ \to \infty$$

Energy cost of FW measurement diverges!

Hotta, Matsumoto and Funo, Phys. Rev. D89, 124023 (2014).



Black hole is formed in the measurement region during the preparation of huge energy for the firewall measurement

$\Rightarrow$ **Firewall Information Censorship**

Informational extension of Cosmic Censorship

M. Hotta, J. Matsumoto and K. Funo, Phys. Rev. D89, 124023 (2014)