MECHANISM

To see the property of $|\psi_k\rangle$,
we take a basis set of energy eigenstates

(although we never employ such a basis
in practical calculations)

$$|\psi_0\rangle = \sum_n c_n |n\rangle \qquad (\hat{h}|n\rangle = e_n|n\rangle)$$

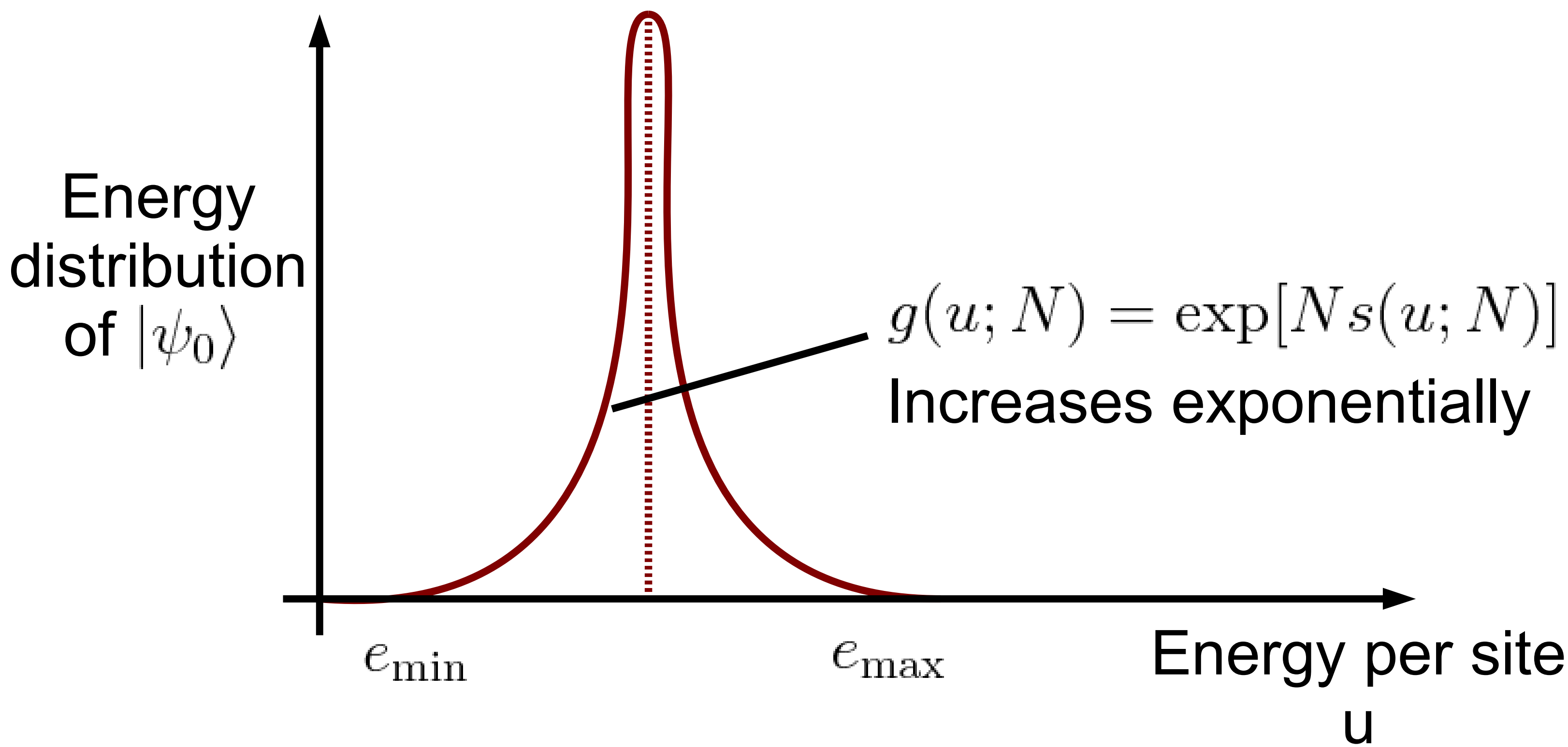
MECHANISM

Energy distribution of $|\psi_0\rangle = \sum_n c_n |n\rangle \quad (\hat{h}|n\rangle = e_n|n\rangle)$

becomes

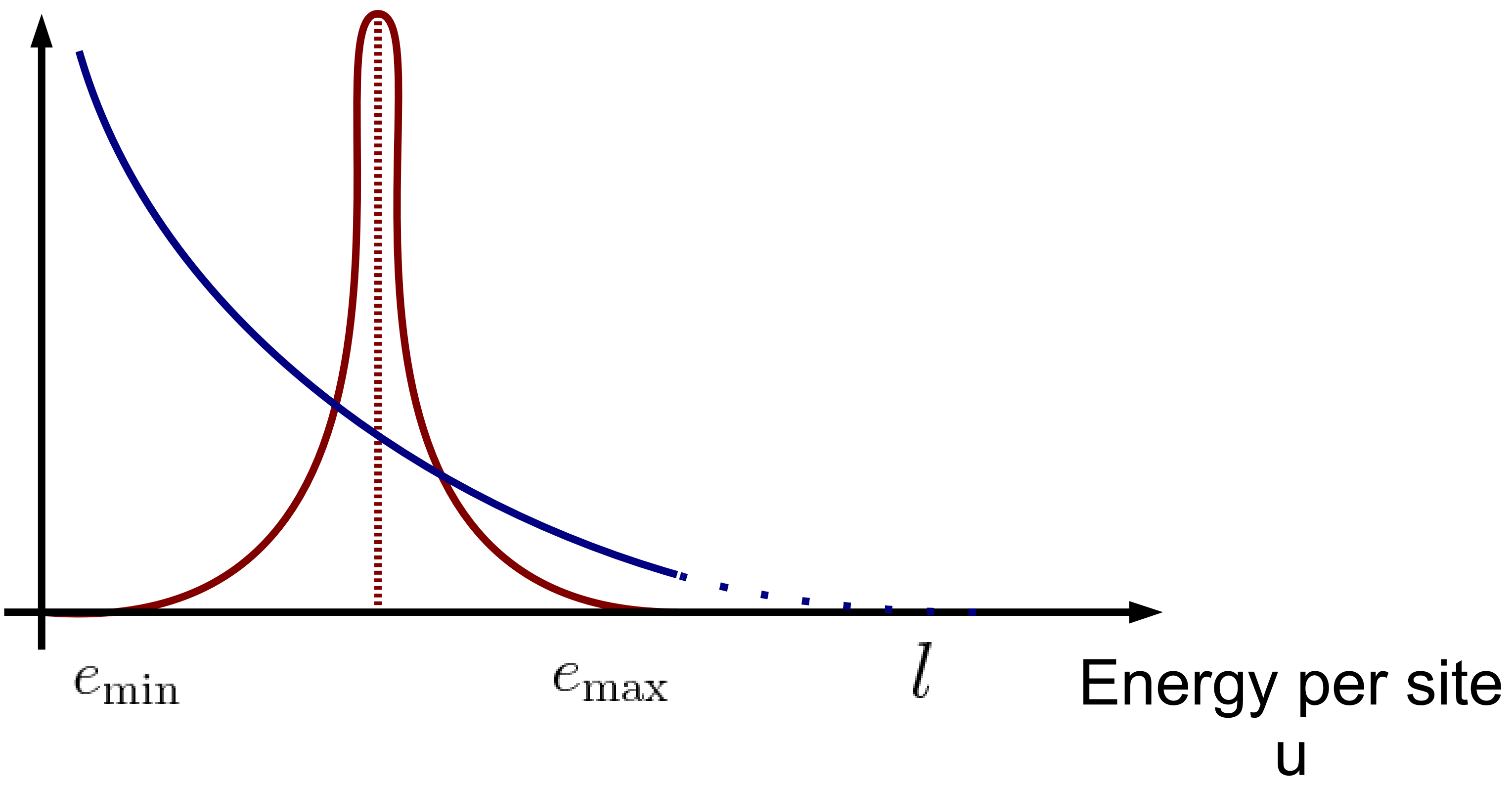
$$\sum_{\substack{n \\ \text{s.t } e_n \in [u-\delta/2,\, u+\delta/2)}}' |c_n|^2 \propto g(u;N)$$

($g(u;N)$: density of states)



$$|\psi_k\rangle \propto (l-\hat{h})^k|\psi_0\rangle$$
$$= \sum_n c_n (l-e_n)^k |n\rangle$$

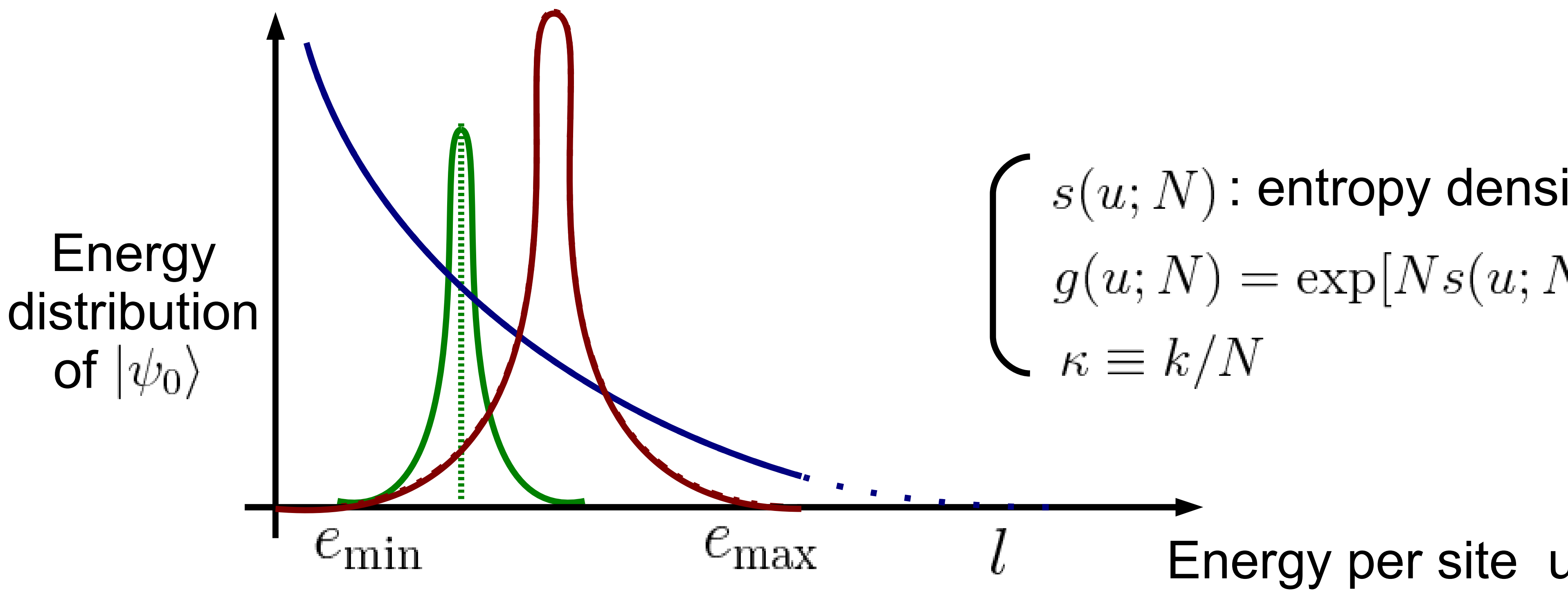
$(\, l > \|\hat{h}\| \,)$



Energy distribution of $|\psi_k\rangle \propto (l-\hat{h})^k|\psi_0\rangle$ becomes

$$\left(|\psi_0\rangle = \sum_n c_n|n\rangle \quad (\hat{h}|n\rangle = e_n|n\rangle) \right)$$

$$\begin{aligned} \sum_n{}' \quad |c_n|^2(l-e_n)^{2k} &\propto g(u;N)(l-u)^{2k} \\ &= \exp[N\{s(u;N)+2\kappa\ln(l-u)\}] \end{aligned}$$



$$\left(\begin{array}{l} s(u;N) \text{ : entropy density} \\ g(u;N) = \exp[Ns(u;N)] \\ \kappa \equiv k/N \end{array} \right)$$

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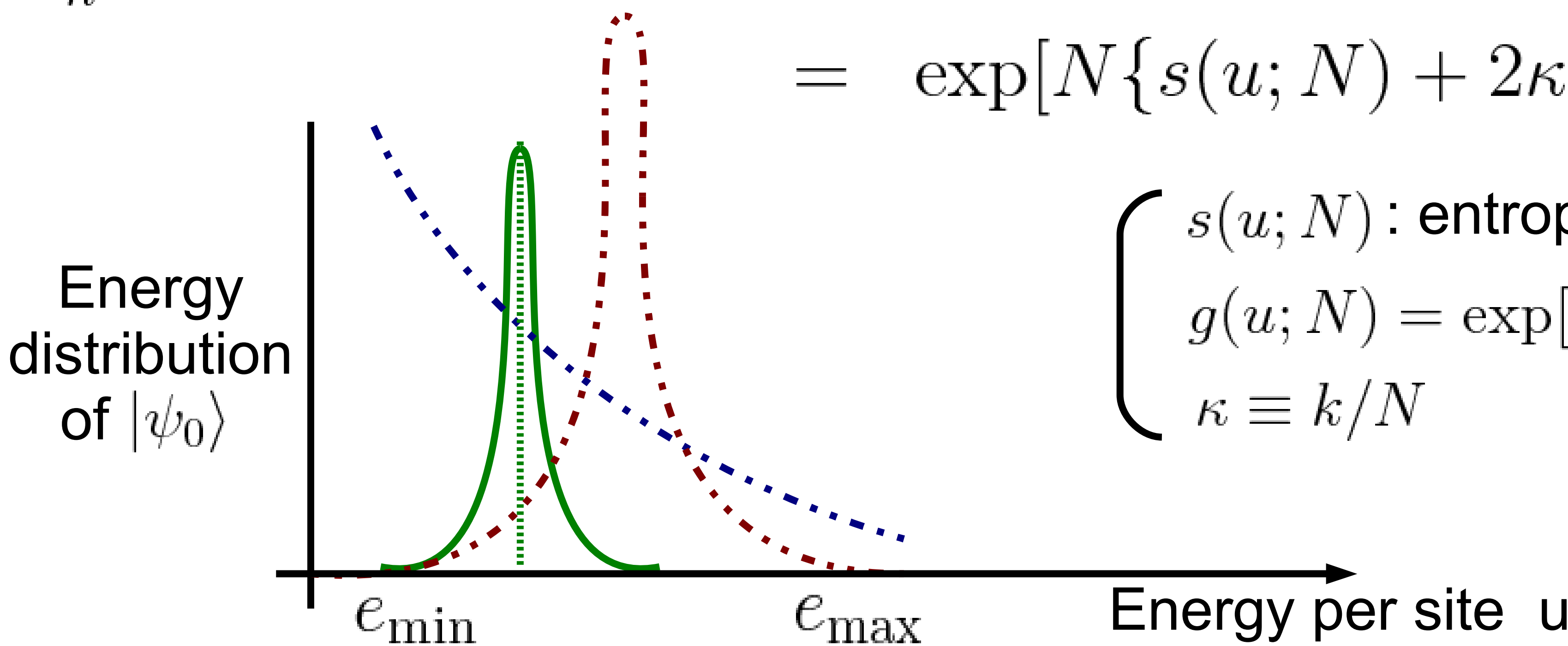

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We manipulate a superposition of unknown energy eigenstates by multiplying the polynomial of Hamiltonian.

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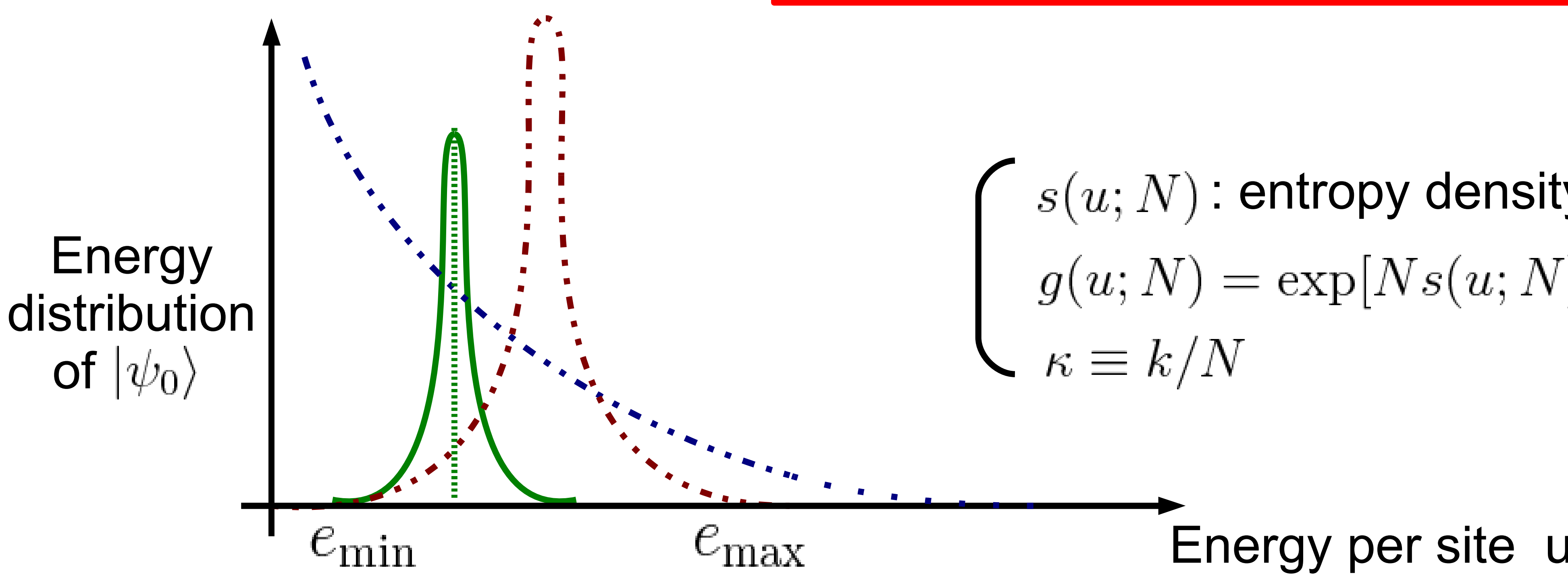

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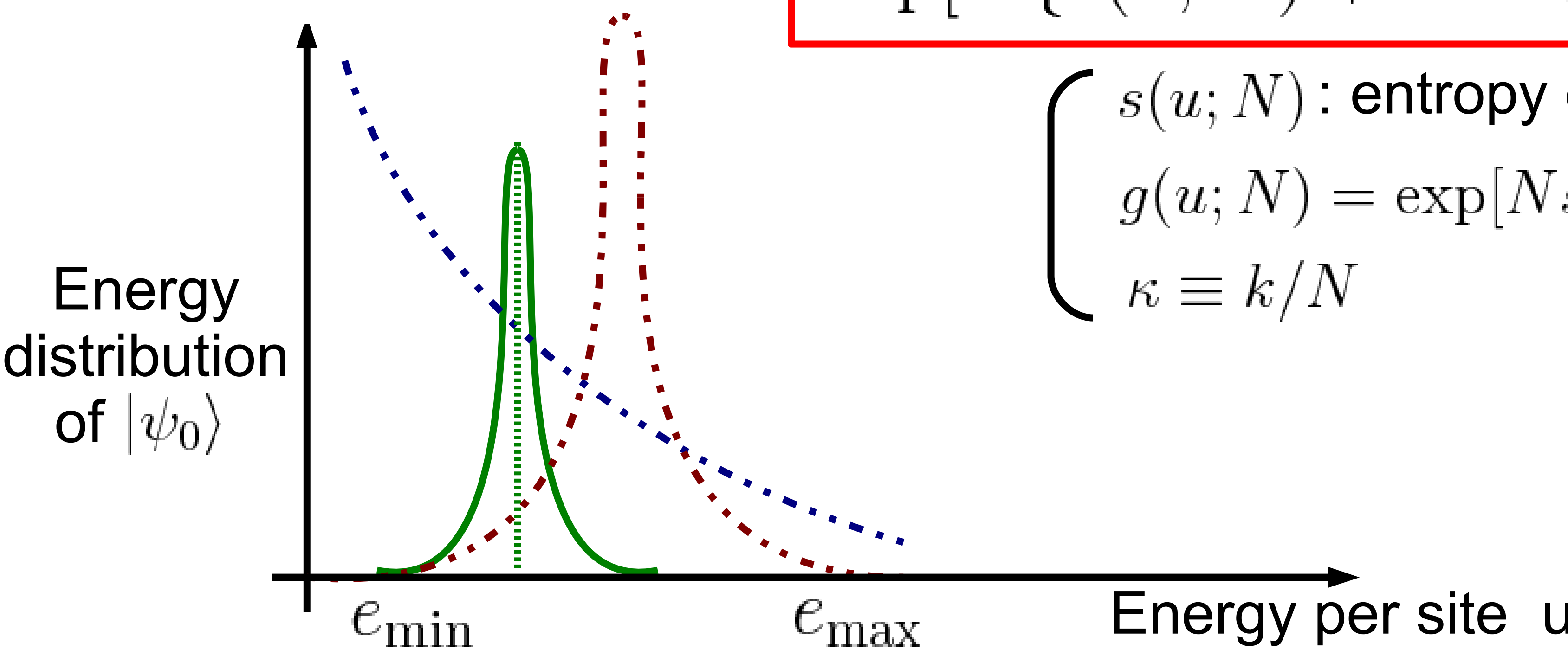

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$\xi_\kappa(u;N) \equiv s(u;N)+2\kappa\ln(l-u)$ takes the maximum at u_κ^* which satisfies

$$\beta(u_\kappa^*;N) = \frac{2\kappa}{(l-u_\kappa^*)}.$$

$$\left(\begin{array}{c} \beta(u;N) \equiv \dfrac{\partial s(u;N)}{\partial u} \\ \kappa \equiv k/N \end{array}\right)$$

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\left(\begin{array}{c} \beta(u;N) \equiv \dfrac{\partial s(u;N)}{\partial u} \\ \kappa \equiv k/N \end{array} \right)
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We get

$$
\xi_\kappa(u;N) = \xi_\kappa(u_\kappa^*;N) - \frac{1}{2}|\xi_\kappa''|(u - u_\kappa^*)^2 + \frac{1}{6}\xi_\kappa'''(u - u_\kappa^*)^3 + \cdots
$$

$$
\left(\xi_\kappa''(u_\kappa^*;N) = \beta'(u_\kappa^*;N) - 2\kappa/(l - u_\kappa^*) \right)
$$

0

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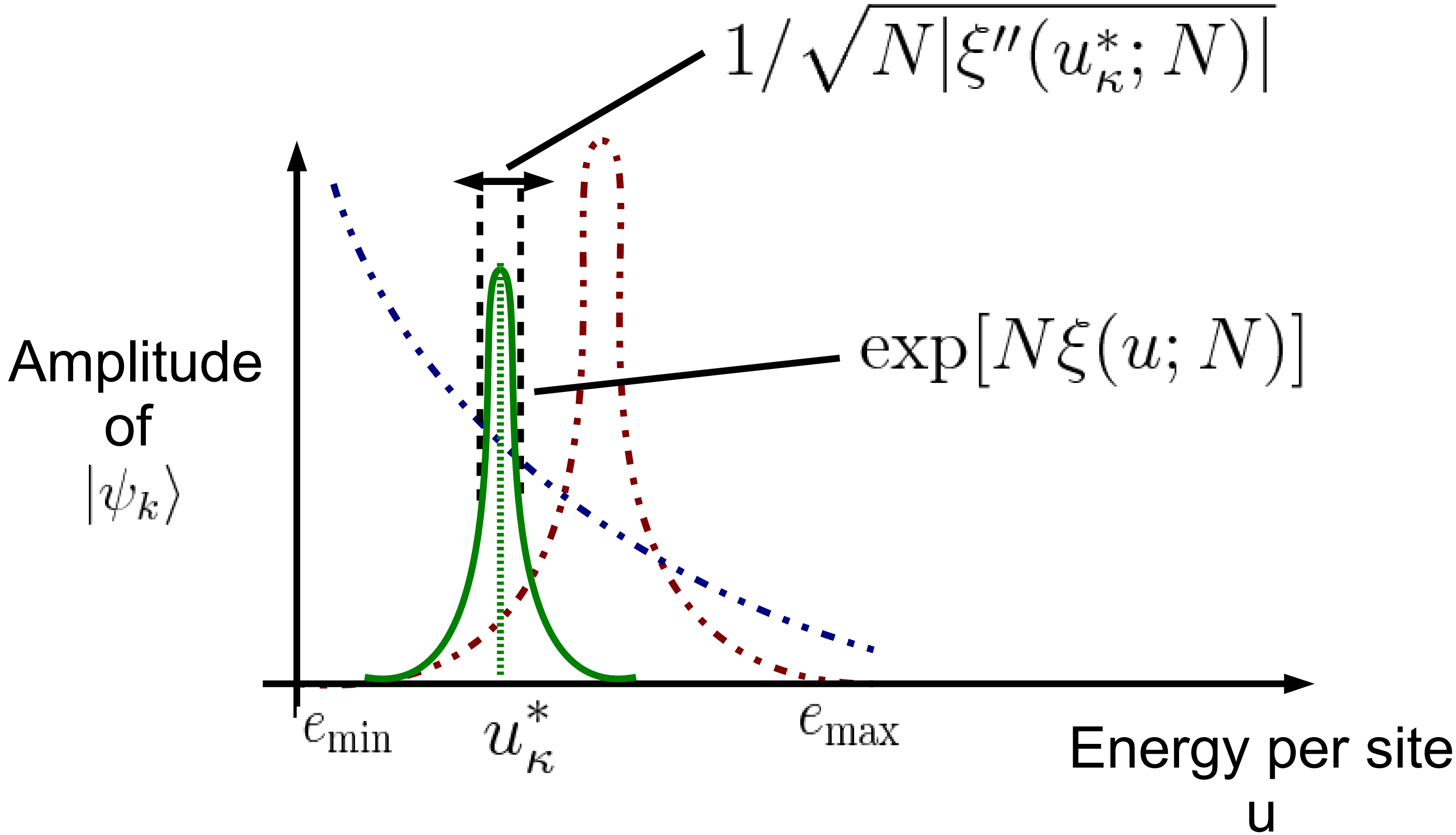

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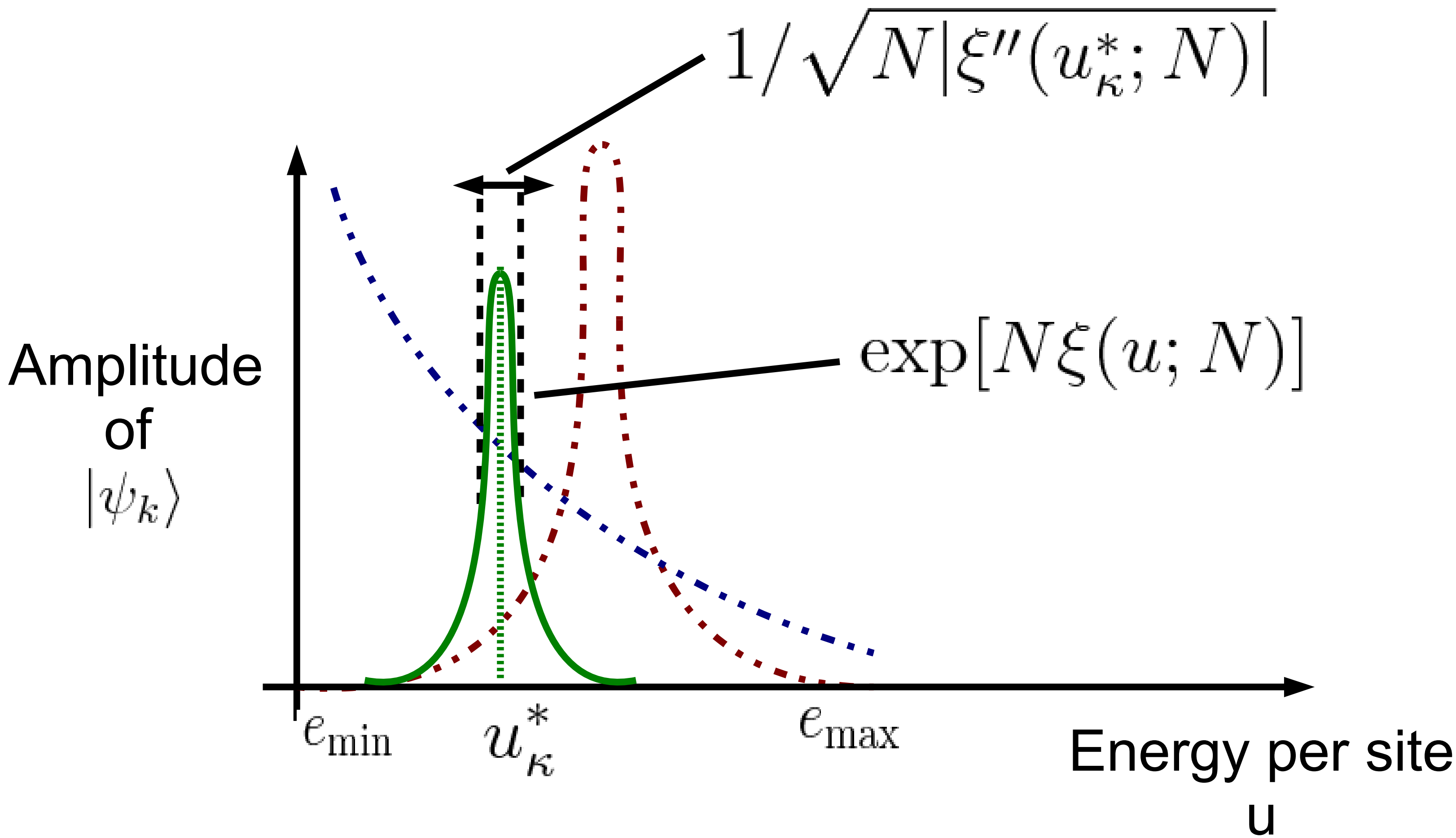

This means that $\langle\psi_k|\hat{A}|\psi_k\rangle$ approaches the microcanonical ensemble average of $\hat{A}$ as $\mathrm{N}\rightarrow\infty$.